Spin transfer from first principles

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1. Landauer-Buttiker formula

2. L-B for spin transfer
   - mixing conductances

3. Spin Torque

4. Spin-pumping and Gilbert damping

5. Conclusion

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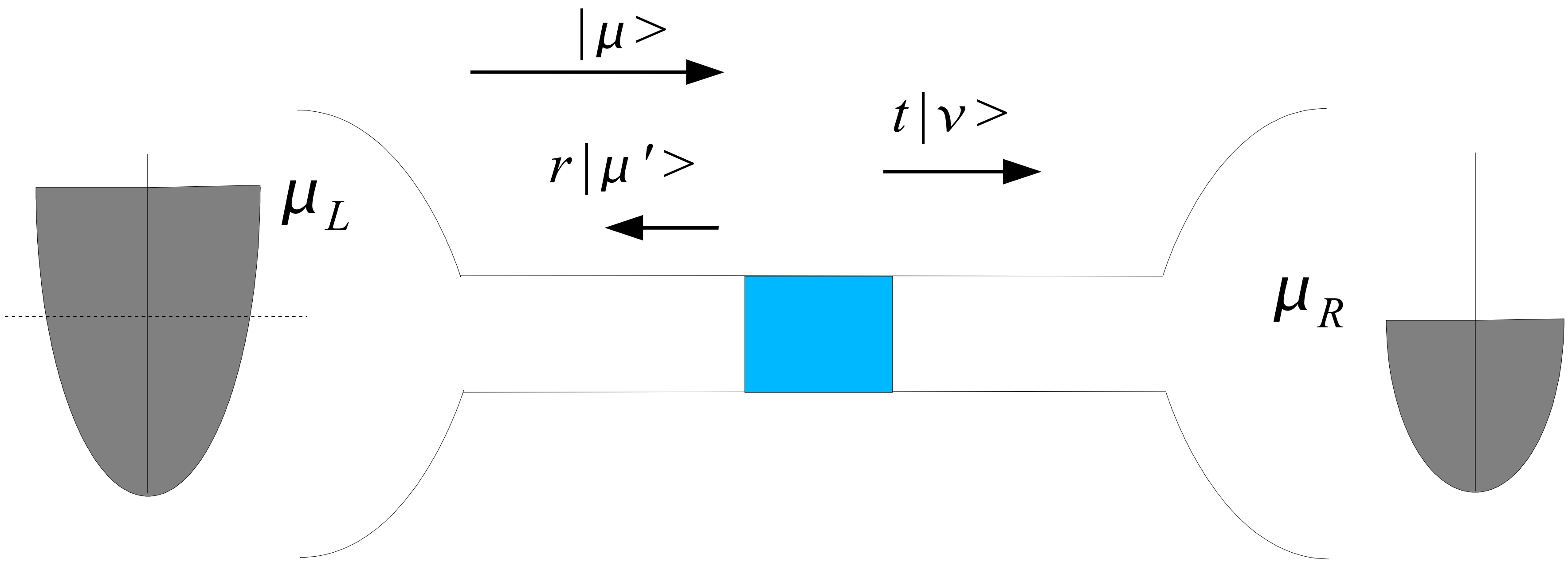
Landauer-Büttiker formula

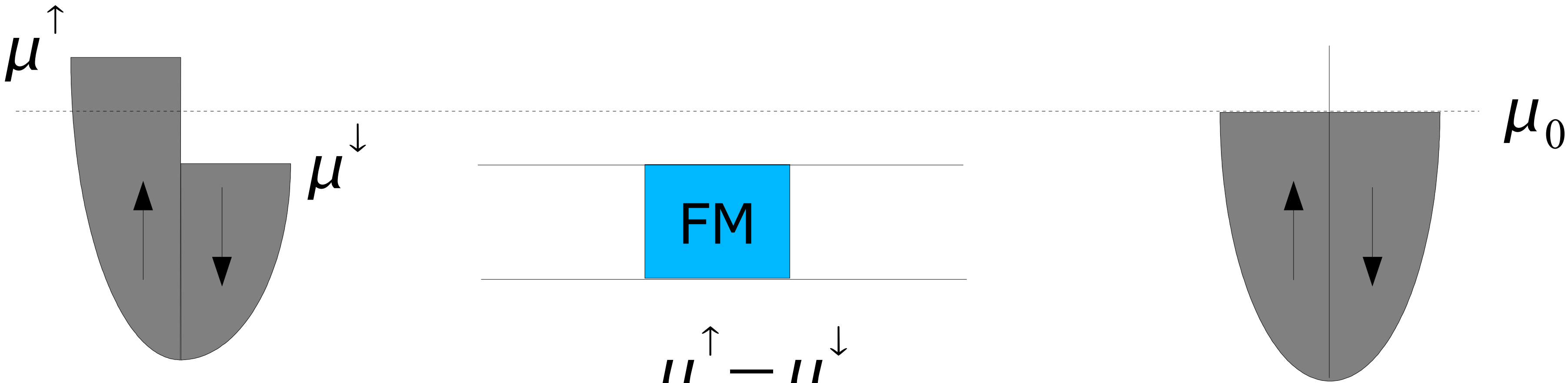


$$I_p = \frac{1}{2\pi}\int_{E(k)\in(\mu_L,\mu_R)} v(k)\,T\,dk = \frac{1}{h}\int_{\mu_R}^{\mu_L} T(E)\,dE = \frac{1}{h}T(E_F)\,\Delta\mu$$

$$v(k) = \frac{1}{\hbar}\frac{\partial E}{\partial k}$$

$$G = \frac{1}{h}\sum_{\mu\nu}\left|t_{\nu\mu}\right|^2$$

“Spin bias”



$$\mu_s = \frac{\mu^{\uparrow} - \mu^{\downarrow}}{2}$$

$$I_p^{\uparrow} = \frac{1}{h} T^{\uparrow} \mu_s \qquad I_p^{\downarrow} = -\frac{1}{h} T^{\downarrow} \mu_s$$

$$I_s = \frac{\hbar}{2} I_p^{\uparrow} - \frac{\hbar}{2} I_p^{\downarrow} = \frac{1}{2\pi} \frac{T^{\uparrow} + T^{\downarrow}}{2} \mu_s$$

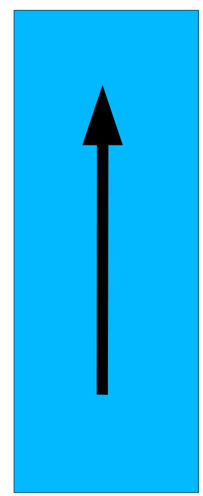
Non-collinear situation

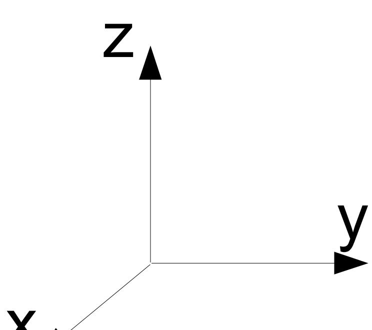
$$|\Psi\rangle^{\text{in}} = |\uparrow\rangle_x = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_z + |\downarrow\rangle_z\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}_z$$

$$|\Psi\rangle^{\text{in}}$$

$$|\Psi\rangle^{\text{t}} = \frac{1}{\sqrt{2}}\begin{pmatrix}t^{\uparrow}\\t^{\downarrow}\end{pmatrix}_z$$

$$|\Psi\rangle^{\text{r}} = \frac{1}{\sqrt{2}}\begin{pmatrix}r^{\uparrow}\\r^{\downarrow}\end{pmatrix}_z$$





Polarization

Left side:

$$\left\langle \Psi^{\mathrm{r}} \middle| \sigma_x \middle| \Psi^{\mathrm{r}} \right\rangle = \frac{1}{2}\left(r^{\uparrow *}, r^{\downarrow *} \right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} r^{\uparrow} \\ r^{\downarrow} \end{pmatrix}$$

$$= \frac{1}{2}\left(r^{\uparrow *} r^{\downarrow} + r^{\downarrow *} r^{\uparrow} \right) = \mathrm{Re}\left(r^{\uparrow} r^{\downarrow *} \right)$$

$$\left\langle \Psi^{\mathrm{r}} \middle| \sigma_y \middle| \Psi^{\mathrm{r}} \right\rangle = -\mathrm{Im}\left(r^{\uparrow} r^{\downarrow *} \right)$$

$$\left\langle \Psi^{\mathrm{r}} \middle| \sigma_z \middle| \Psi^{\mathrm{r}} \right\rangle = \frac{1}{2}\left(|r^{\uparrow}| - |r^{\downarrow}| \right)$$

Right side:

$$\left\langle \Psi^{\mathrm{t}} \middle| \sigma_x \middle| \Psi^{\mathrm{t}} \right\rangle = \mathrm{Re}\left(t^{\uparrow} t^{\downarrow *} \right)$$

$$\left\langle \Psi^{\mathrm{t}} \middle| \sigma_y \middle| \Psi^{\mathrm{t}} \right\rangle = -\mathrm{Im}\left(t^{\uparrow} t^{\downarrow *} \right)$$

$$\left\langle \Psi^{\mathrm{t}} \middle| \sigma_z \middle| \Psi^{\mathrm{t}} \right\rangle = \frac{1}{2}\left(|t^{\uparrow}| - |t^{\downarrow}| \right)$$

$$\left|\Psi\right\rangle^{\mathrm{in}} = \left|\downarrow\right\rangle_x = \frac{1}{\sqrt{2}}\left(-\left|\uparrow\right\rangle_z + \left|\downarrow\right\rangle_z\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}_z$$

$$\left|\Psi\right\rangle^{\mathrm{r}} = \frac{1}{\sqrt{2}}\begin{pmatrix} -r^{\uparrow} \\ r^{\downarrow} \end{pmatrix}_z \qquad \left|\Psi\right\rangle^{\mathrm{t}} = \frac{1}{\sqrt{2}}\begin{pmatrix} -t^{\uparrow} \\ t^{\downarrow} \end{pmatrix}_z$$

Left side:

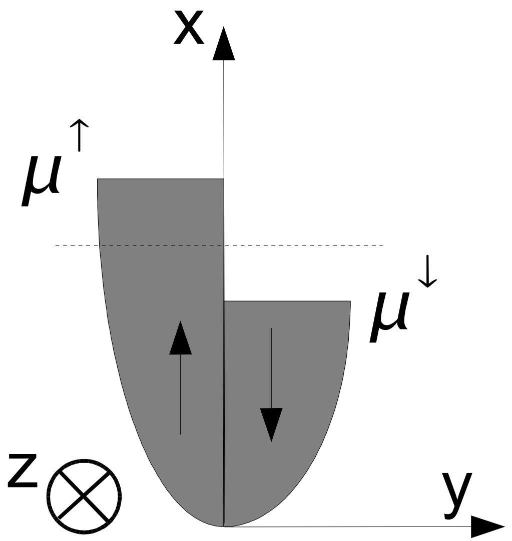
Right side:

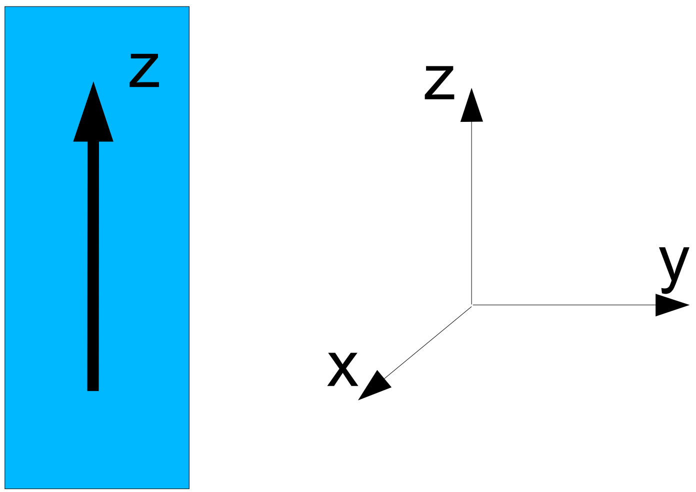
$$\left\langle \Psi^{\mathrm{r}}\right|\sigma_x\left|\Psi^{\mathrm{r}}\right\rangle = -\mathrm{Re}\left(r^{\uparrow} r^{\downarrow *}\right) \qquad = -\mathrm{Re}\left(t^{\uparrow} t^{\downarrow *}\right)$$

$$\left\langle \Psi^{\mathrm{r}}\right|\sigma_y\left|\Psi^{\mathrm{r}}\right\rangle = \mathrm{Im}\left(r^{\uparrow} r^{\downarrow *}\right) \qquad = \mathrm{Im}\left(t^{\uparrow} t^{\downarrow *}\right)$$

$$\left\langle \Psi^{\mathrm{r}}\right|\sigma_z\left|\Psi^{\mathrm{r}}\right\rangle = \frac{1}{2}\left(|r^{\uparrow}| - |r^{\downarrow}|\right) \qquad = \frac{1}{2}\left(|t^{\uparrow}| - |t^{\downarrow}|\right)$$

The distribution of carriers





$$\vec{I}_s^{\text{in}} = \frac{1}{2\pi} \begin{pmatrix} N \\ 0 \\ 0 \end{pmatrix} \mu_s = \frac{1}{2\pi} N \vec{\mu}_s$$

$$\vec{\mu}_s = \begin{pmatrix} (\mu^{\uparrow} - \mu^{\downarrow})/2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mu_s \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{I}_s^{\text{r}} = -\frac{1}{2\pi} \begin{pmatrix} \mathrm{Re}(r^{\uparrow} r^{\downarrow *}) \\ -\mathrm{Im}(r^{\uparrow} r^{\downarrow *}) \\ 0 \end{pmatrix} \mu_s$$

$$\vec{I}_s^{\text{t}} = \frac{1}{2\pi} \begin{pmatrix} \mathrm{Re}(t^{\uparrow} t^{\downarrow *}) \\ -\mathrm{Im}(t^{\uparrow} t^{\downarrow *}) \\ 0 \end{pmatrix} \mu_s$$

General orientation

$$\vec{I}_s^{\mathrm{in}} = \frac{1}{2\pi} N \vec{\mu}_s$$

$$\vec{I}_s^{\mathrm{r}} = -\frac{1}{2\pi} \begin{pmatrix} \mathrm{Re}\left(r^{\uparrow} r^{\downarrow *}\right) & \mathrm{Im}\left(r^{\uparrow} r^{\downarrow *}\right) & 0 \\ -\mathrm{Im}\left(r^{\uparrow} r^{\downarrow *}\right) & \mathrm{Re}\left(r^{\uparrow} r^{\downarrow *}\right) & 0 \\ 0 & 0 & \left(R^{\uparrow} + R^{\downarrow}\right)/2 \end{pmatrix} \vec{\mu}_s$$

$$\vec{I}_s^{\mathrm{t}} = \frac{1}{2\pi} \begin{pmatrix} \mathrm{Re}\left(t^{\uparrow} t^{\downarrow *}\right) & \mathrm{Im}\left(t^{\uparrow} r^{\downarrow *}\right) & 0 \\ -\mathrm{Im}\left(t^{\uparrow} r^{\downarrow *}\right) & \mathrm{Re}\left(t^{\uparrow} r^{\downarrow *}\right) & 0 \\ 0 & 0 & \left(T^{\uparrow} + T^{\downarrow}\right)/2 \end{pmatrix} \vec{\mu}_s$$

The conductances

"Standard" L-B conductances:

$$g^{\uparrow} = \sum_{\mu\nu} \left| t^{\uparrow}_{\mu\nu} \right|^2 = \sum_{\mu\mu'} \left(\delta_{\mu\mu'} - \left| r^{\uparrow}_{\mu\mu'} \right|^2 \right) \qquad g^{\downarrow} = \sum_{\mu\nu} \left| t^{\downarrow}_{\mu\nu} \right|^2$$

$$g^{\mathrm{Sh}} = \sum_{\mu\nu} \delta_{\mu\mu'} = N$$

Mixing (complex) conductances :

$$g_r^{\uparrow\downarrow} = \sum_{\mu\mu'} \left(\delta_{\mu\mu'} - r^{\uparrow}_{\mu\mu'} r^{\downarrow *}_{\mu\mu'} \right) \qquad g_t^{\uparrow\downarrow} = \sum_{\mu\nu} t^{\uparrow}_{\mu\nu} t^{\downarrow *}_{\mu\nu}$$

Spin circuit theory (G.E.W. Bauer, A.Brataas, Y.Nazarov .....)

Spin torque:

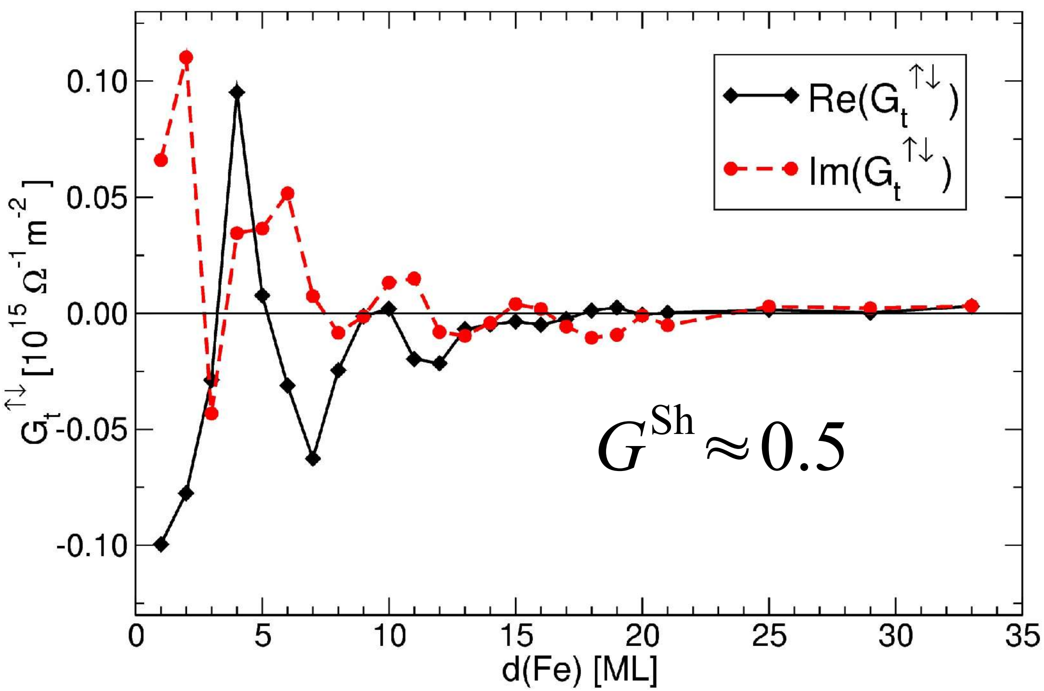
$$\mathrm{Torque} = \vec{I}_s^{\,\mathrm{in}} - \vec{I}_s^{\,\mathrm{out}}$$

J.C. Slonczewski, JMMM **159**, L1 (1996)
L. Berger, PRB **54**, 9353 (1996)

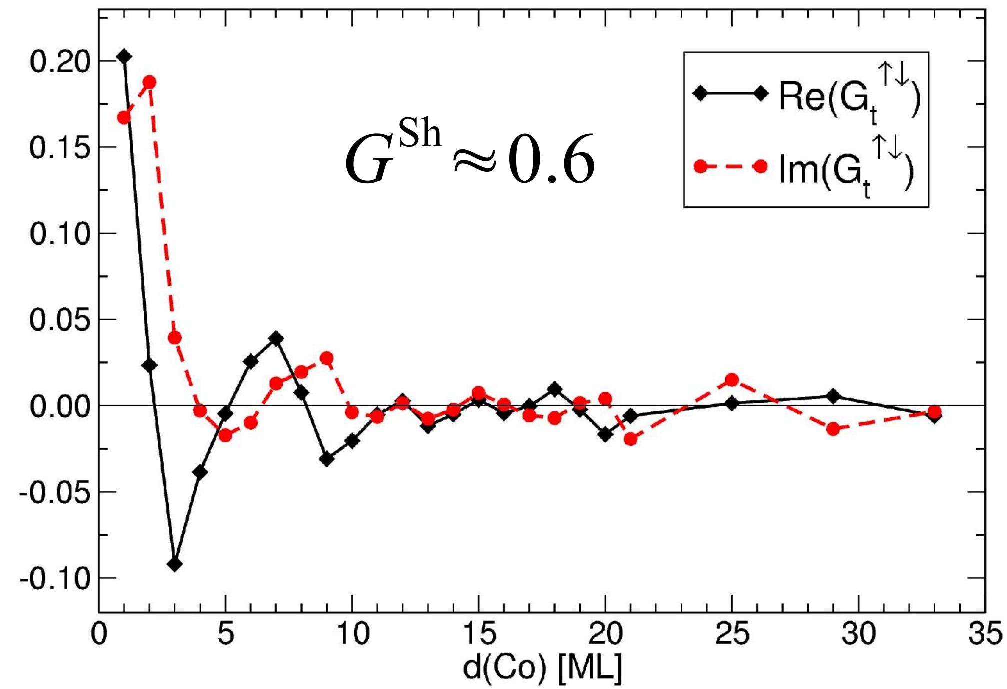
J.A.Katine *et. al.* PRL **84**, 3149 (2000)

Transmission mixing conductance

Au/Fe(d ML)/Au(001)



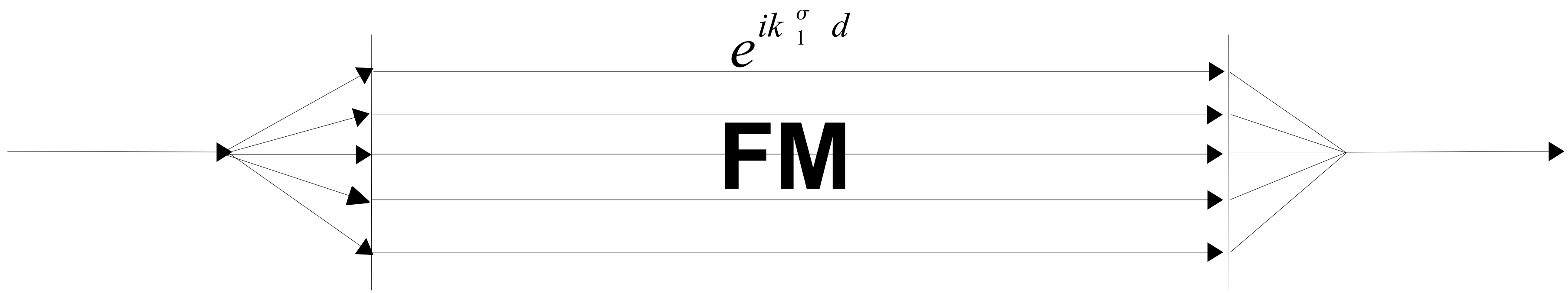
Cu/Co(d ML)/Cu(111)



The amplitude of the transmitted spin current drops to less than 10% already after several ML

Position-dependent precession

M.D. Stiles, A.Zangwill, PRB **66,** 014407 (2002)



$$t^\sigma_{\mathrm{N}\to\mathrm{F}} = \begin{pmatrix} t_1^\sigma \\ \vdots \\ t_n^\sigma \end{pmatrix} \qquad \Lambda^\sigma = \begin{pmatrix} e^{ik_1^\sigma d} & \cdots & 0 \\ 0 & \ddots & 0 \\ 0 & \cdots & e^{ik_n^\sigma d} \end{pmatrix} \qquad t^\sigma_{\mathrm{F}\to\mathrm{N}} = \left(t_1^\sigma, \ldots, t_n^\sigma\right)$$

$$t^\sigma \approx t^\sigma_{\mathrm{F}\to\mathrm{N}} \Lambda^\sigma t^\sigma_{\mathrm{N}\to\mathrm{F}}$$

$$g_t^{\uparrow\downarrow} = \sum_{\mu\nu} t^\uparrow_{\mu\nu} t^{\downarrow *}_{\mu\nu} = \int dk_\parallel t^\uparrow(k_\parallel) t^{\downarrow *}(k_\parallel) \to \int dk_\parallel e^{i(k_i^\uparrow - k_j^\downarrow)d} \ldots$$

Stationary points: $\nabla_{k_\parallel}(k_i^\uparrow - k_j^\downarrow) = 0$

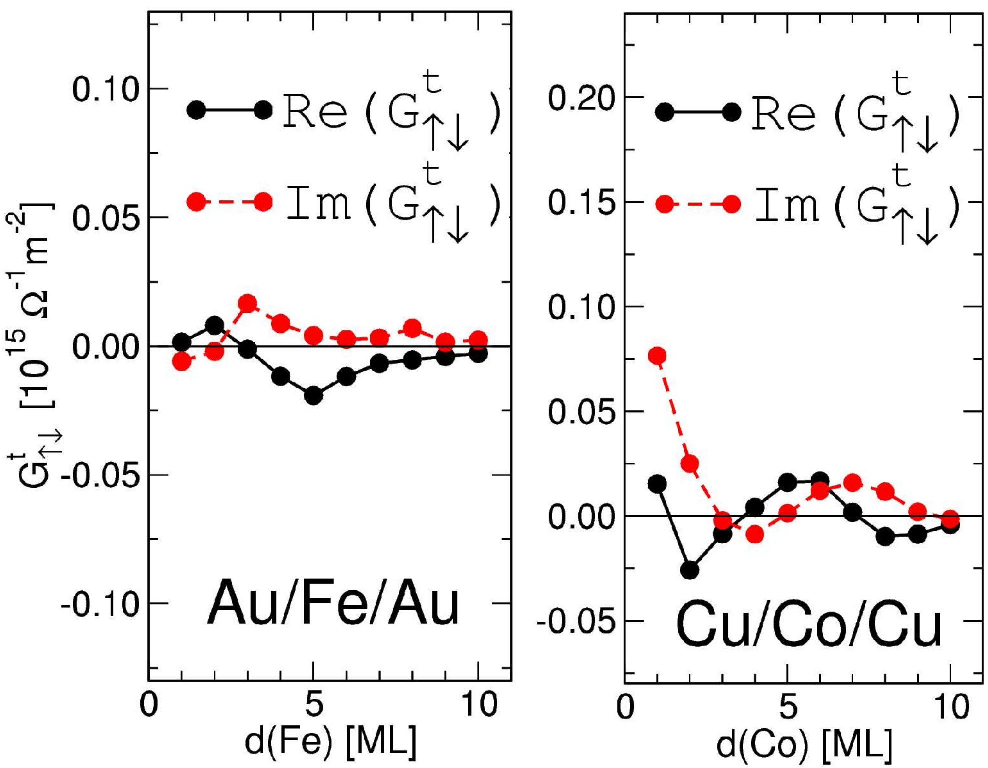
Free electrons:

$$\sim \frac{k_F^\uparrow k_F^\downarrow}{|k_F^\uparrow - k_F^\downarrow|} \frac{e^{i\left(k_F^\uparrow - k_F^\downarrow\right)d}}{d}$$

IEC: P.Bruno, PRB **52**, 411 (95)

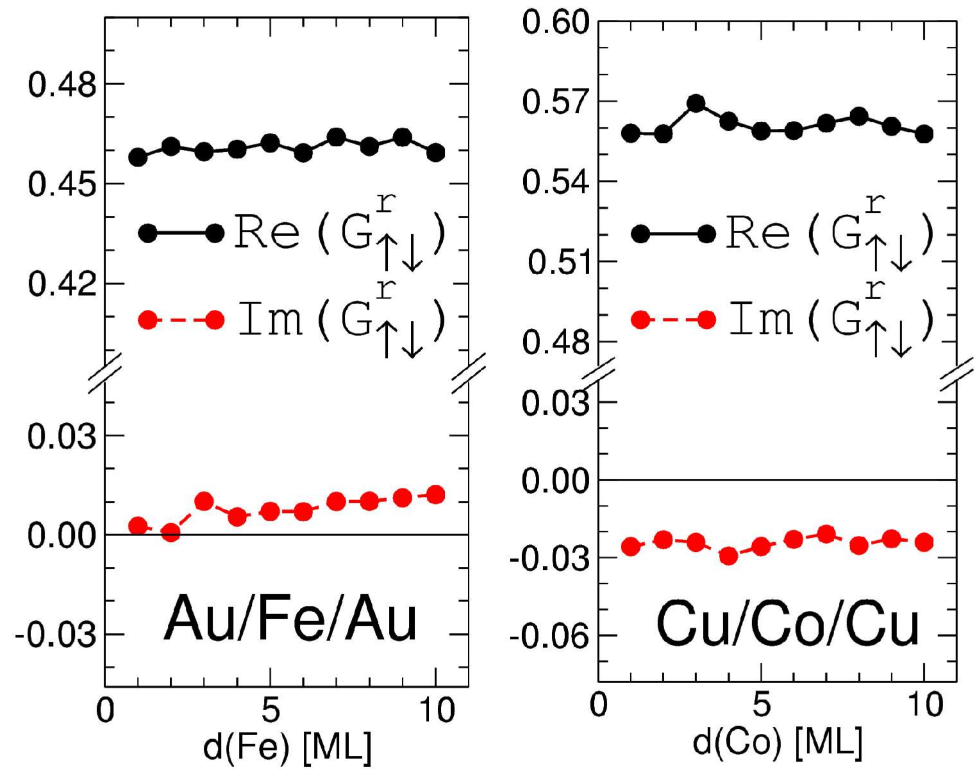
Disordered interfaces (50% alloy)

$$g_t^{\uparrow\downarrow} = \sum_{\mu\nu} t_{\mu\nu}^{\uparrow} t_{\mu\nu}^{\downarrow *}$$



$$g_t^{\uparrow\downarrow} \approx 0$$

$$g_r^{\uparrow\downarrow} = \sum_{\mu\mu'} \left(\delta_{\mu\mu'} - r_{\mu\mu'}^{\uparrow} r_{\mu\mu'}^{\downarrow *} \right)$$



$$\mathrm{Im}\, g_r^{\uparrow\downarrow} \approx 0$$

$$\mathrm{Re}\, g_r^{\uparrow\downarrow} \approx N = g^{\mathrm{Sh}}$$

Gilbert damping

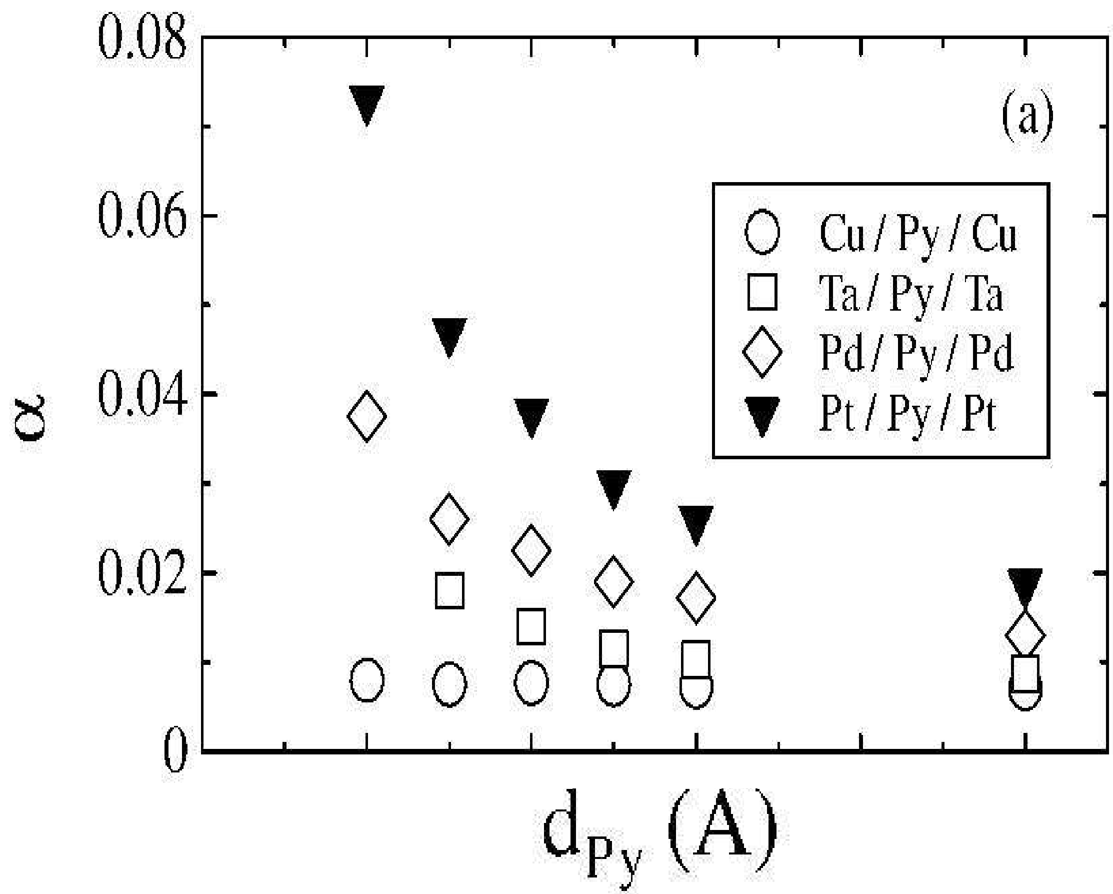
Landau-Lifshitz-Gilbert equation:

$$\frac{d\boldsymbol{m}}{dt} = -\gamma\,\boldsymbol{m} \times \boldsymbol{H}_{eff} + \alpha\,\boldsymbol{m} \times \frac{d\boldsymbol{m}}{dt}$$

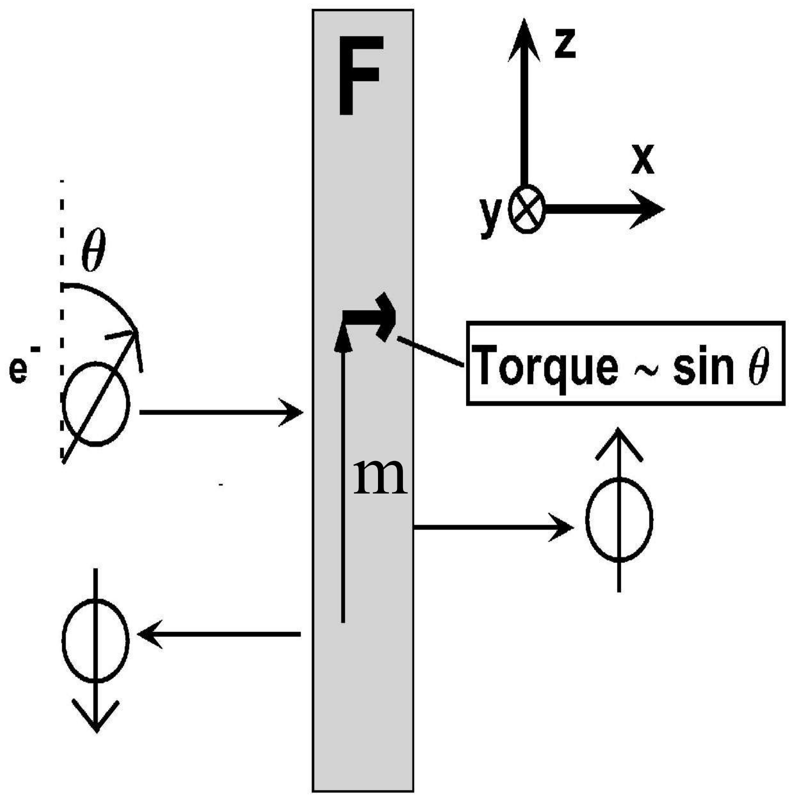
The values of α parameter can be **enhanced by two orders of magnitude** for thin layers

E.B. Myers *et al.*, 1999;
C.H. Back *et al.*, 1999
R. Urban *et al.*, 2001;
S. Mizukami *et al.*, 2001

S. Mizukami *et al.*, 2001



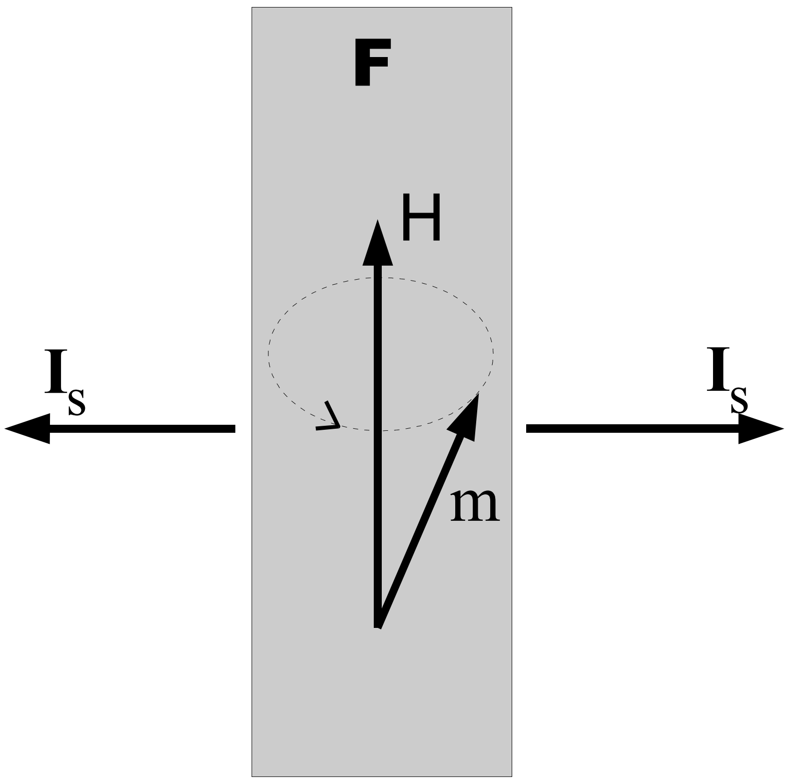
Current-induced magnetization reversal



X.Waintal *et al.* 2000

Spin pumping

Y.Tserkovnyak, A. Brataas, G. E. W. Bauer, PRL **88,** 117601 (2002)



$$\boldsymbol{I}_s^{pump} = \frac{\hbar}{4\pi}\left(A_r \; \boldsymbol{m} \times \frac{d\,\boldsymbol{m}}{dt} - A_i \; \frac{d\,\boldsymbol{m}}{dt} \right)$$

$$\frac{d\,\boldsymbol{m}}{dt} = -\gamma_0 \boldsymbol{m} \times \boldsymbol{H}_{\boldsymbol{eff}} + \alpha_0 \boldsymbol{m} \times \frac{d\,\boldsymbol{m}}{d\,t} + \frac{\gamma_0}{M} \boldsymbol{I}_s^{pump}$$

$$\frac{d\,\boldsymbol{m}}{dt} = -\gamma\, \boldsymbol{m} \times \boldsymbol{H}_{\boldsymbol{eff}} + \alpha\, \boldsymbol{m} \times \frac{d\,\boldsymbol{m}}{d\,t}$$

$$\frac{1}{\gamma} = \frac{1}{\gamma_0}\left\{ 1 + g_L A_i / 4\pi M \right\}$$

$$A_r + iA_i = \left(g_{\mathrm{r}}^{\uparrow\downarrow} - g_{\mathrm{t}}^{\uparrow\downarrow} \right) S$$

$$\alpha = \frac{\gamma}{\gamma_0}\left\{ \alpha_0 + g_L A_r / 4\pi M \right\}$$

we now have:

$$A_r + iA_i \longrightarrow A_r \approx \mathrm{Re}\, g^{\uparrow\downarrow} S, \quad A_i \approx 0$$

and:

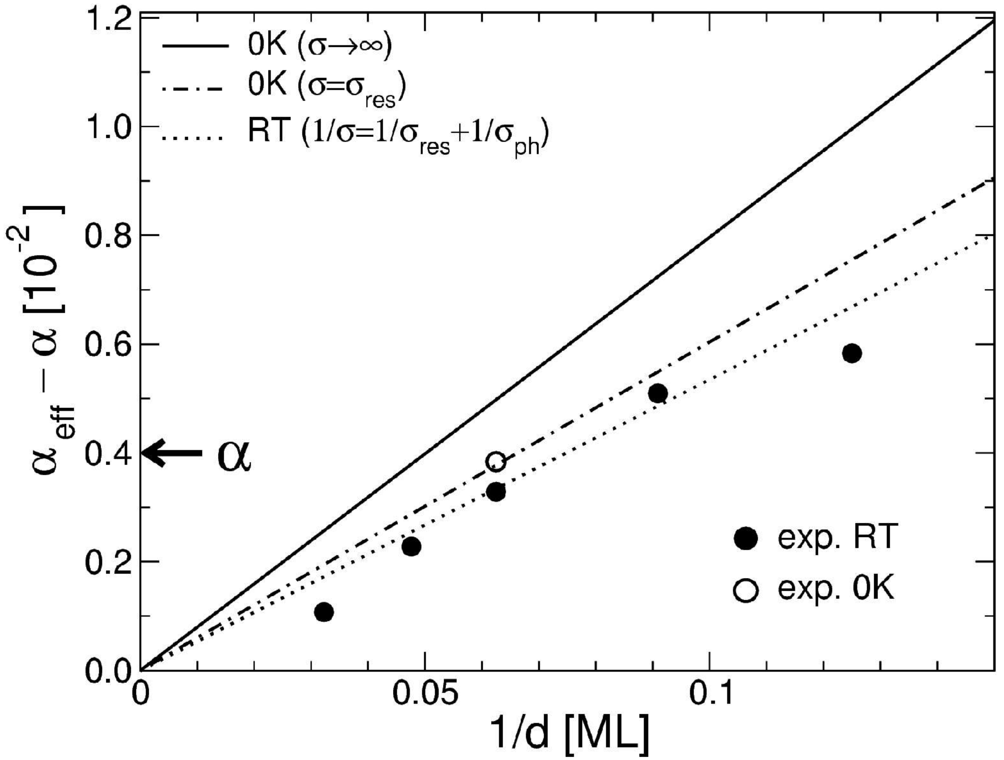
$$\frac{1}{\gamma} = \frac{1}{\gamma_0}\left\{1 + g_L A_i / 4\pi M\right\} \longrightarrow \gamma = \gamma_0$$

$$\alpha = \frac{\gamma}{\gamma_0}\left\{\alpha_0 + g_L A_r / 4\pi M\right\} \longrightarrow \alpha = \alpha_0 + \alpha'$$

$$\alpha' = g_L \mathrm{Re}\, g^{\uparrow\downarrow} S / 4\pi M \longrightarrow \boxed{\alpha' \sim \frac{\mathrm{Re}(g^{\uparrow\downarrow})}{d}}$$

FMR in Fe/Au (001) multilayers

R.Urban et al., PRL **87** (2001)



$$\sigma_{ph} = 0.45 \times 10^{8}\, \Omega^{-1} m^{-1}$$

$$\sigma_{res} = 0.24 \times 10^{8}\, \Omega^{-1} m^{-1}$$

$$\frac{1}{\tilde{g}_r^{\uparrow\downarrow}} = \frac{1}{g_r^{\uparrow\downarrow}} + \frac{e^2}{h} \frac{L}{\sigma}$$

Y. Tserkovnyak, PRB **66**, 224403 (2002)

Conclusions

- Spin transport can be parametrized using mixing conductances:

$$g_t^{\uparrow\downarrow} = \sum_{\mu\nu} t_{\mu\nu}^{\uparrow} t_{\mu\nu}^{\downarrow *} \qquad g_r^{\uparrow\downarrow} = \sum_{\mu\mu'} \left(\delta_{\mu\mu'} - r_{\mu\mu'}^{\uparrow} r_{\mu\mu'}^{\downarrow *} \right)$$

- Transverse spin current is absorbed within few monolayers of the N/F interface

- Spin-pumping is responsible for the ehanced Gilbert damping observed in thin magnetic layers

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