

# AN ANALYTICALLY EXACT DERIVATION OF THE $g$ -FACTOR OF THE ELECTRON IN THE HYDROGEN ATOM

Jos A.A.J. Perenboom and Charles M.E.E. Peters

High Field Magnet Laboratory, Institute for Molecules and Materials,  
Radboud University, Toernooiveld 7, NL-6525 ED Nijmegen, the Netherlands

In his famous 1933-paper “the Lagrangean in Quantum Mechanics”, Paul Dirac has linked Quantum Mechanics with Hamilton-Jacobi classical mechanics. The 2-particle problem of an electron bound to a proton, the hydrogen atom H, is the only system that one may be able to exactly solve classically. The Lagrangean is a non-relativistic approximation, and here we report a study, using modern computer algebra, of a fully relativistic calculation. The results of our analysis indicate the existence of stable orbits, resulting from a competition of the Coulomb force (attracting as  $1/r^2$ ) and the magnetic interaction between the proton- and electron-spins (repulsive at short distances as  $1/r^{5/2}$ ). We view the result as classically-derived “quantum mechanics”.

The analysis also provides a simple geometric explanation for the  $g$ -factor of the electron to 6 digits. By including an applied magnetic field, the Zeeman effect of the two spin systems could be evaluated in the spherically symmetric, orbital  $S$  ground state: hence we can derive the effective magnetic moments (i.e.  $\mu_B$ ) of electron and proton from first principles. The outcome of the analysis is that we can construct a calculable standard for the magnetic field, and in an experiment fix the strength of the magnetic field in terms of fundamental constants with uncertainties much less than 1 ppm. This could provide a Euromet standard for the SI-Tesla, easily transferred using NMR probes, ensuring magnetic field measurement traceability up to 7 digits or an uncertainty of 0.1 ppm.

Physics of Magnetism'08, 24-27 June 2008, Poznan Poland

Category 2: Quantum and classical spin systems

Presentation: ORAL

Correspondence:

dr. J. Perenboom  
Radboud University, High Field Magnet Laboratory  
Toernooiveld 7  
NL-6525 ED Nijmegen  
the Netherlands  
J.Perenboom@science.ru.nl  
+31 24 3653370

9.7 cm