

Double quantum dot Andreev molecules: numerical phase diagrams, critical evaluation of effective models, and identification of microscopic mechanisms

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We systematically investigate the phase diagram of a parallel double-quantum-dot Andreev molecule, where the two quantum dots are coupled to a common superconducting lead. Using the numerical renormalization group method, we map out the evolution of the ground state across a wide parameter space of level detunings, size of the superconducting gap, lead couplings, and interdot coupling strength. The intricate phase diagrams feature singlet, doublet, and a relatively uncommon triplet ground states, with the latter being a distinct signature of strong lead-mediated interactions between the quantum dots.

We benchmark the applicability of simplified effective models, including the atomic limit and zero-bandwidth approximations, in capturing the complex behavior of this parallel configuration. Our analysis reveals severe limitations of these models, underscoring the necessity for maximal caution when extrapolating beyond their tested validity. In particular, all effective models except for the extended version of the zero-bandwidth approximation failed in reproducing the triplet ground state and made several false predictions [1].

We qualitatively as well as quantitatively recover the NRG results for half-filled dots by straightforward, yet quite tedious 4th order perturbative calculation in the dot-SC-lead tunnel coupling. The analysis of the analytical approach reveals the microscopic mechanism behind the numerical findings as well as the causes of the failure of the simplified effective models [2]. These findings provide crucial insights for interpreting experimental observations and designing superconducting devices based on quantum-dot architectures.

References:

[1] Peter Zalom, Kacper Wrzeźniewski, Tomáš Novotný, and Ireneusz Weymann, “Double-quantum-dot Andreev molecules: Phase diagrams and critical evaluation of effective models”, *Phys. Rev. B* 110, 134506 (2024)

[2] Sachin Verma, Peter Zalom, and Tomáš Novotný, “Singlet-triplet quantum phase transition in a half-filled double quantum dot Andreev molecule”, in preparation