Exact valence bond ground states with and without translational invariance

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Magnetically frustrated systems, while often governed by simple Hamiltonians, exhibit complex behavior and novel phases absent in unfrustrated ones. They are also particularly difficult to study numerically as well as theoretically. However, exact results can be obtained in specific situations. One of which is the Majumdar-Ghosh (MG) model, a special case of the Heisenberg chain with the antiferromagnetic nearest neighbor $J_1 > 0$ and next-to-nearest neighbor $J_2 > 0$ interactions, where $J_2/J_1 = 1/2$. Frustration caused by competition of the two interactions causes formation of valence bonds (dimers). Dimers take a role of building blocks for the doubly degenerate ground state and high portion of excited states. High energy required to break dimers make the state highly resistant to perturbations. MG model is not the only one to host valence bond ground state, in fact it generalizes to a class of linear exchange models.

In our work, we show that the valence bond ground state is supported in an even larger class of Hamiltonians that includes models with broken translational invariance, anizotropy or 3-site and 4-site interactions. Combining DMRG with analytical methods we investigate the spin gap for different Hamiltonians of this exact dimer ground state family. Venturing out of the exact dimerisation we also analyze phase diagram of the $J_1 - J_2 - J_3 - J_4$ model and the $J_1 - J_2$ model phase diagram. We consider generalisations to models with easy-axis anisotropy using $q-$deformation by extending the ordinary $SU(2)$ symmetry to a quantum group.

Studying this class of systems lays the groundwork for understanding novel magnetic materials as well as exploration of possibilities for quantum information systems realized with valence bond states.