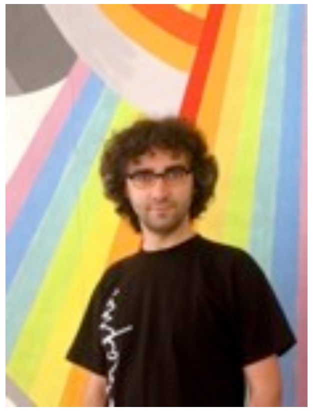
Plasmons in interacting arrays of metallic nanoparticles

Guillaume Weick
Institut de Physique et Chimie des Matériaux de Strasbourg
Université de Strasbourg & CNRS

NanoCTM network meeting, Wąsowo, Sept. 23-27

Collaborators



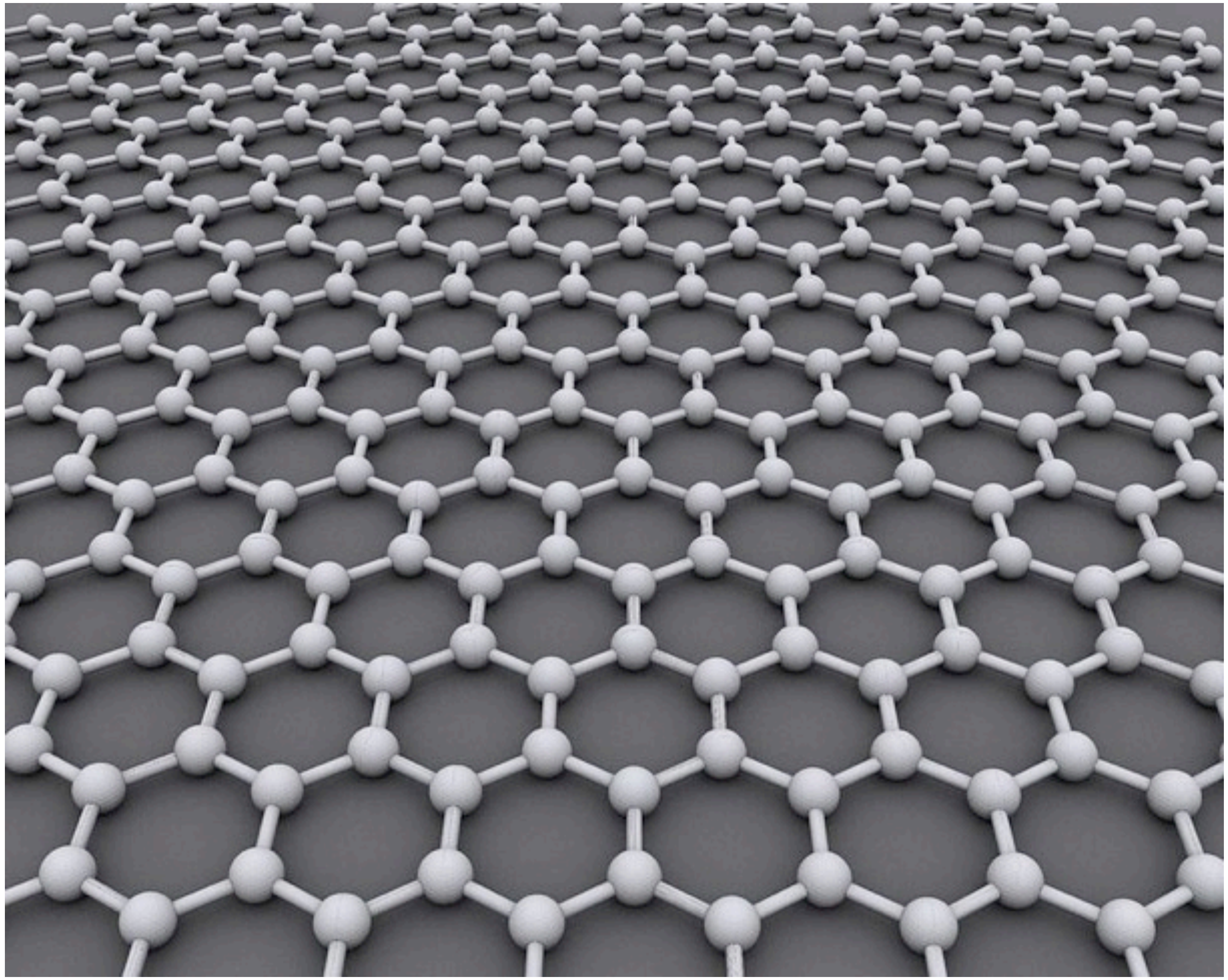
Eros Mariani (University of Exeter)

Claire Woollacott (University of Exeter)

Bill Barnes (University of Exeter)

Ortwin Hess (Imperial College London)

Graphene



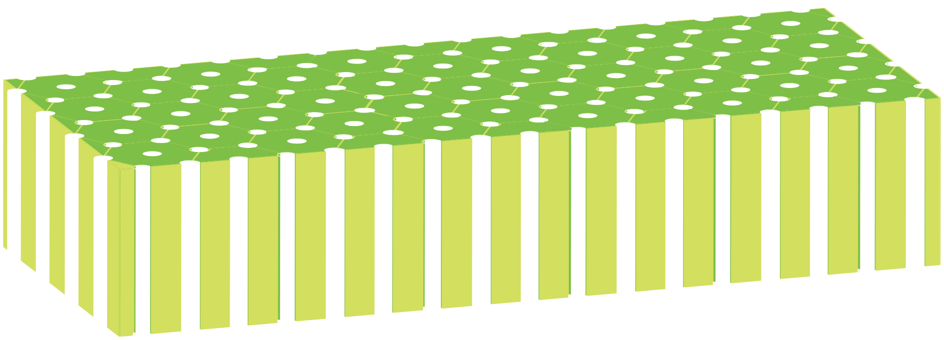
electrons behave as massless Dirac fermions

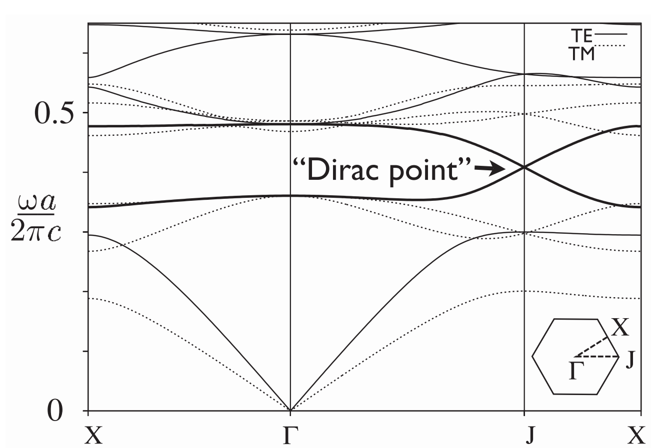
(honeycomb lattice + Bloch theorem)

Artificial graphene

Photonic crystals

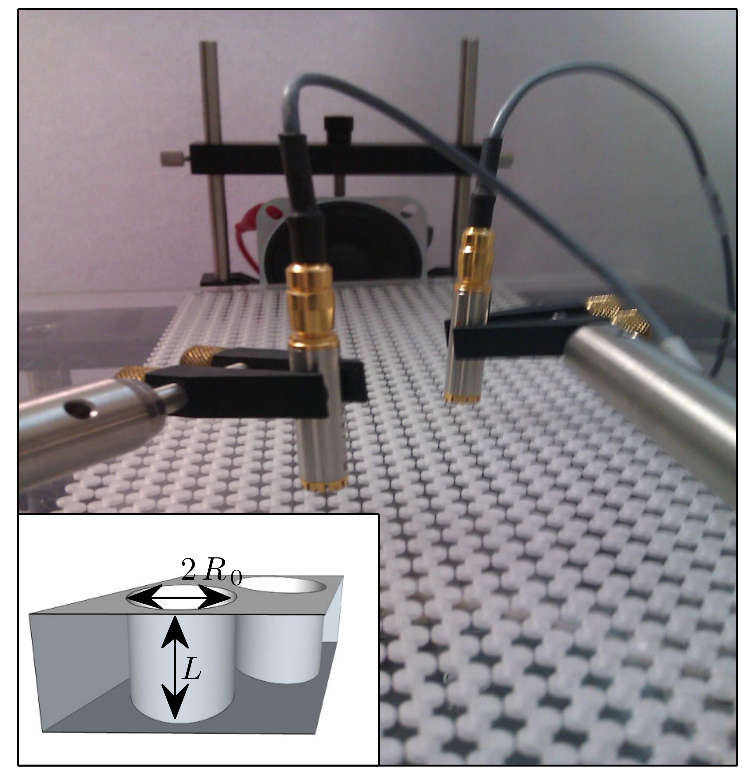
Haldane & Raghu, PRL 2008
Sepkhanov, Bazaliy, Beenakker, PRA 2007





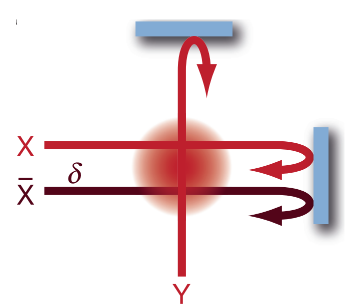
Acoustic waves

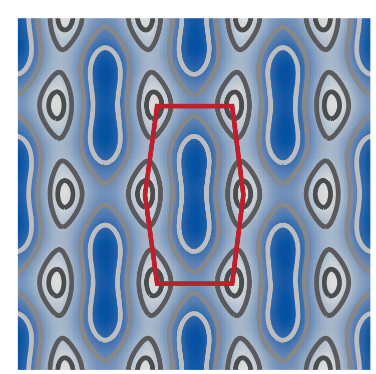
Torrent & Sánchez-Dehesa, PRL 2012

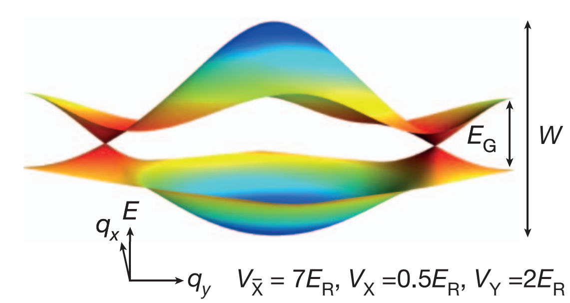


Cold atoms

Esslinger group, Nature 2012





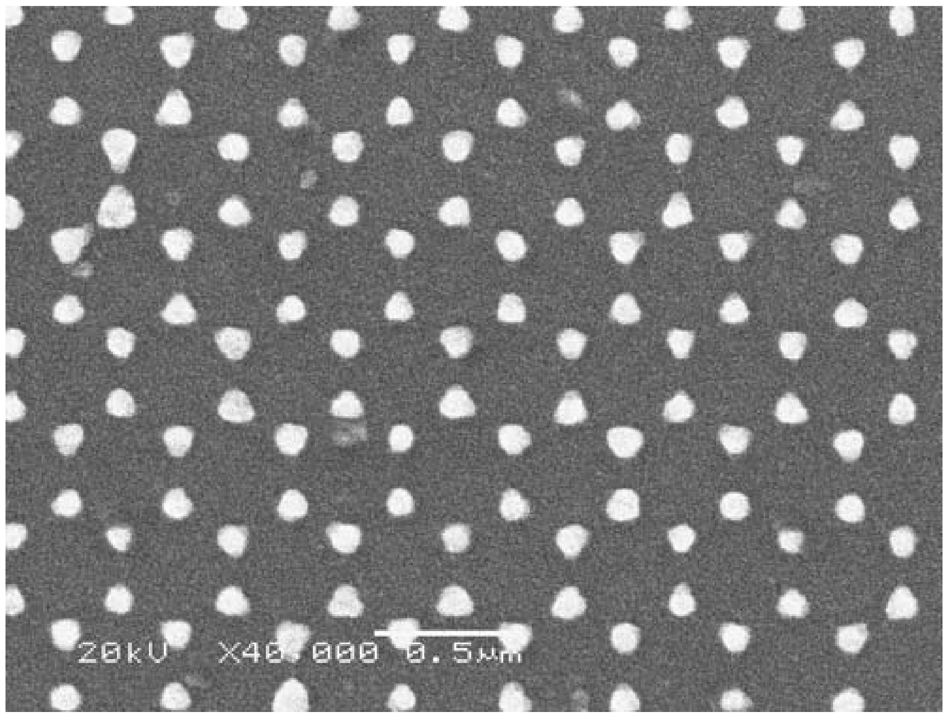


Review: Polini *et al.*, Nature Nanotech. 2013

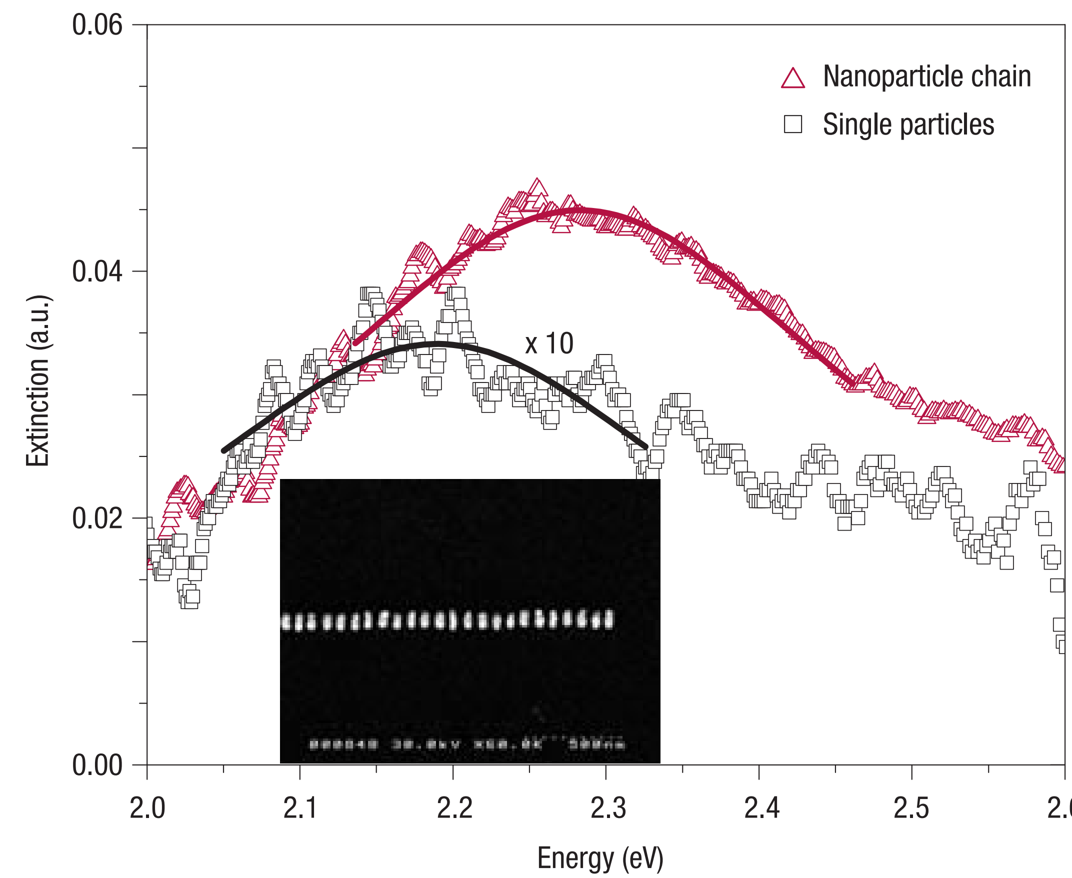
Plasmonic analogue of graphene

UNIVERSITÉ DE STRASBOURG

Plasmonic metamaterial



Zhu *et al.*, Plasmonics 2009
Han *et al.*, PRL 2009



Maier *et al.*, Nature Materials 2003

<u>individual nanoparticle</u>
localized surface plasmon

→

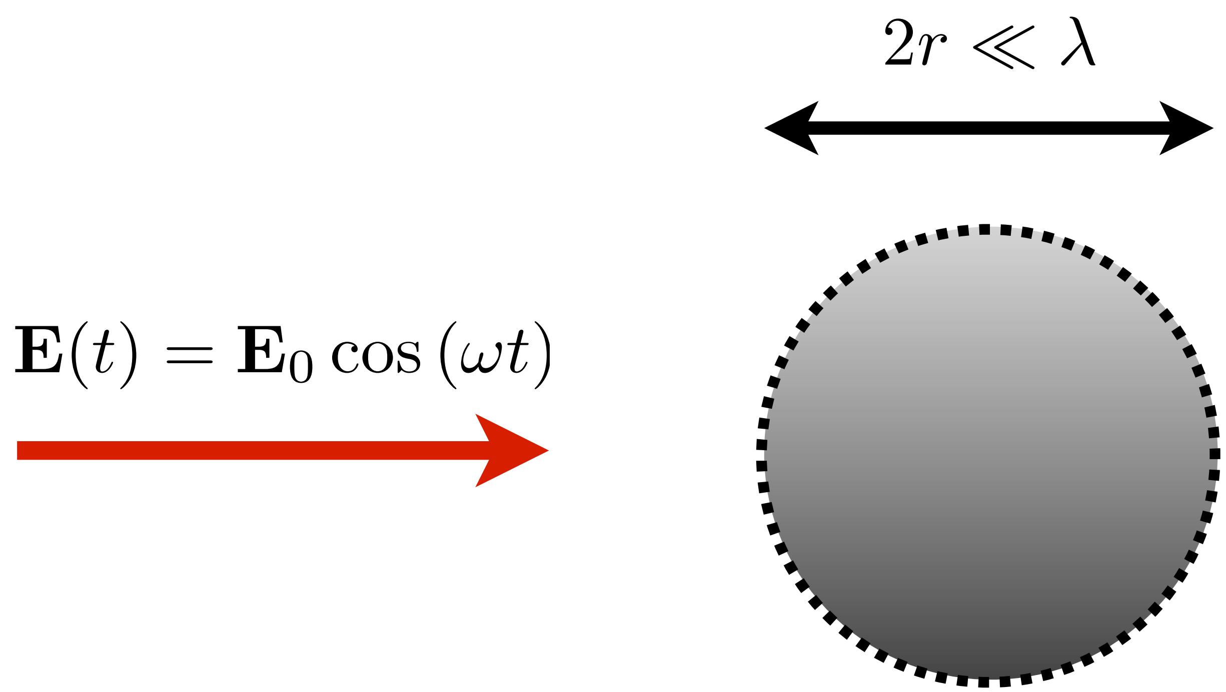
<u>nanoparticle array</u>
collective plasmon
(can propagate over macroscopic distances)

- subwavelength optics
- plasmonic "circuitry"

Individual nanoparticle

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass



(oscillating dipole moment)

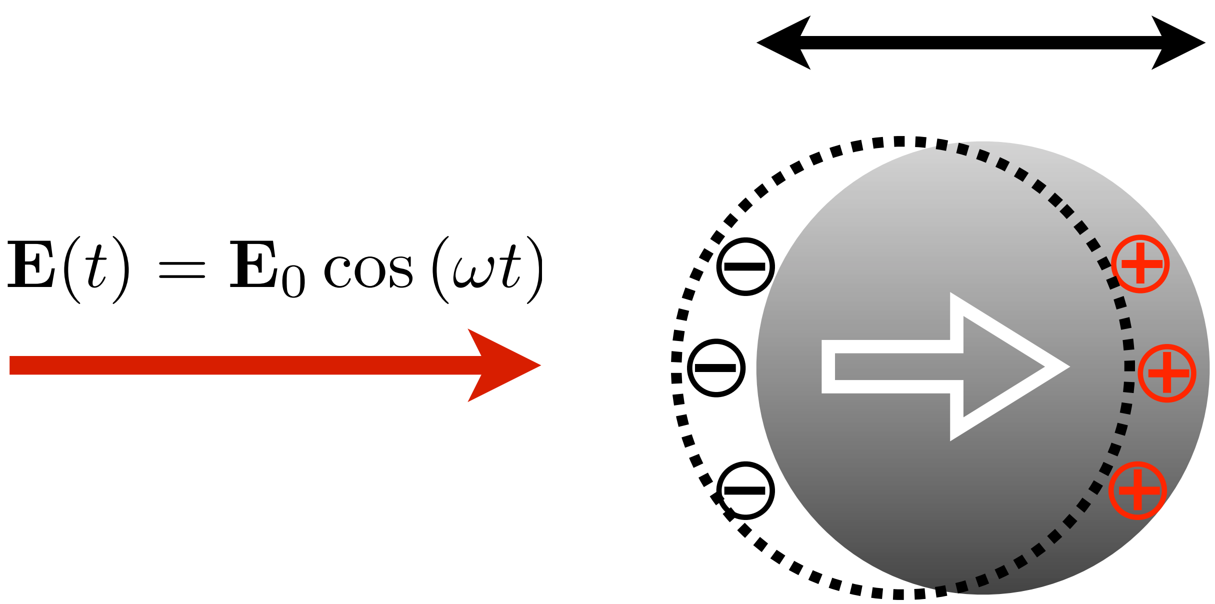
Individual nanoparticle

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass

$$2r \ll \lambda$$

$$\mathbf{E}(t) = \mathbf{E}_0 \cos{(\omega t)}$$

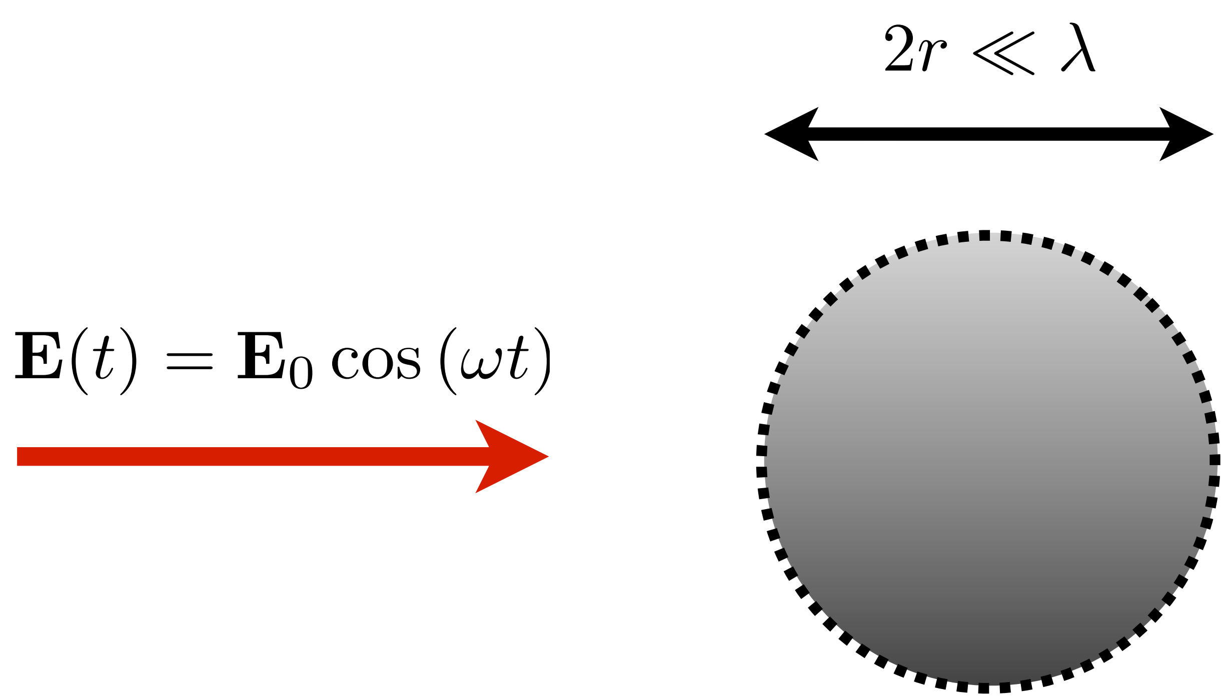


(oscillating dipole moment)

Individual nanoparticle

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass

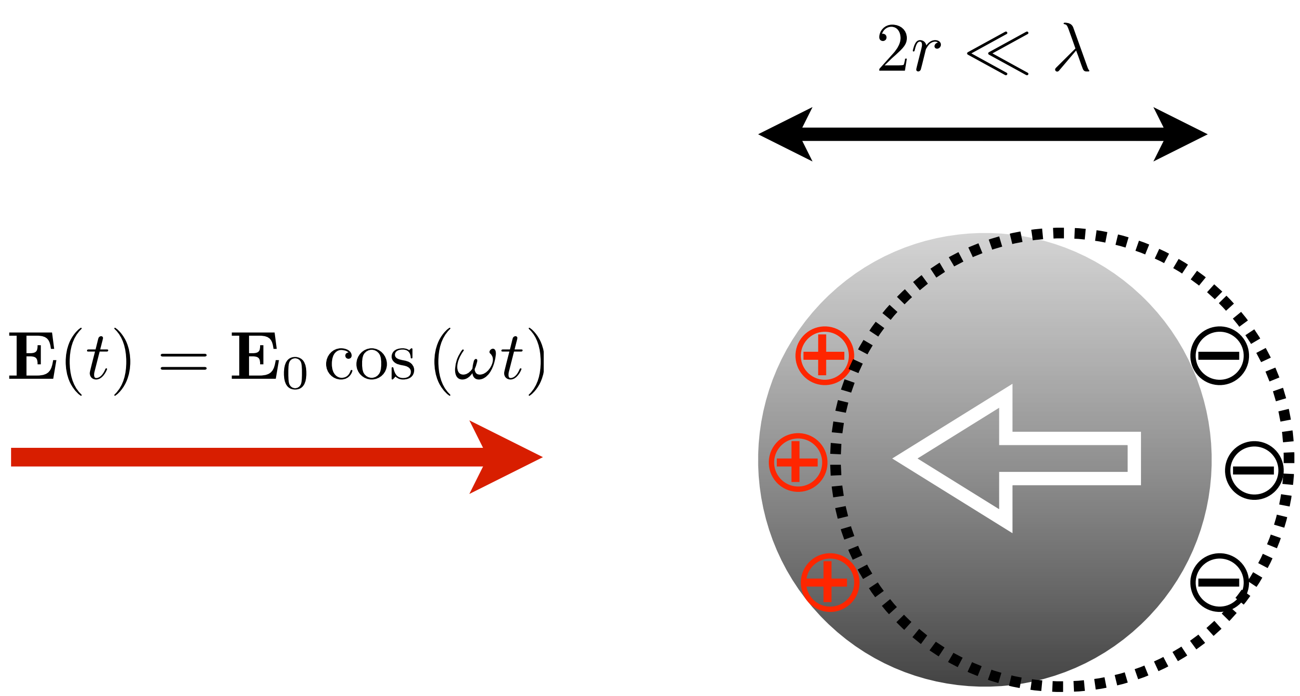


(oscillating dipole moment)

Individual nanoparticle

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass



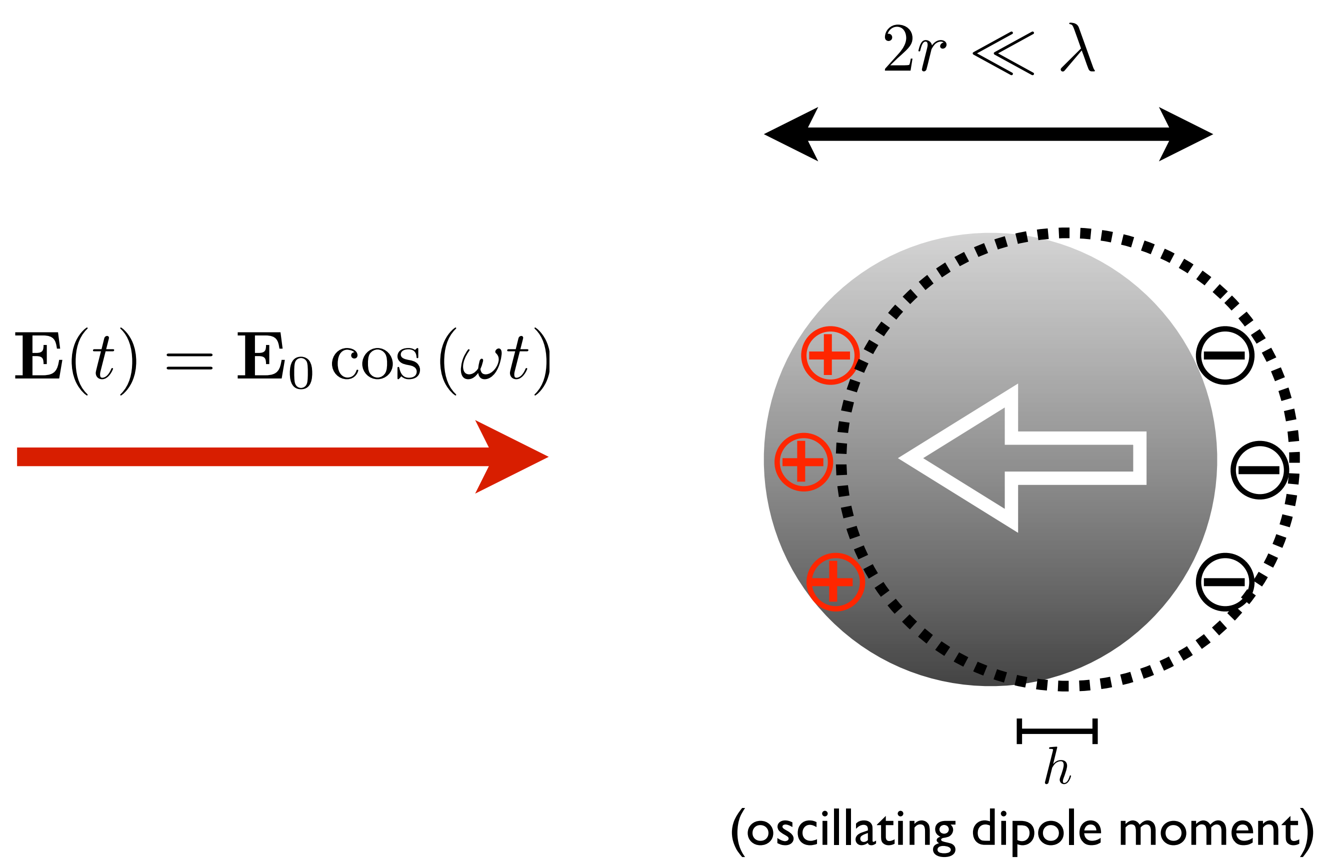
(oscillating dipole moment)

Individual nanoparticle

UNIVERSITÉ DE STRASBOURG

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass



$$H_{\mathrm{LSP}} = \frac{\Pi^2}{2M} + \frac{M}{2}\omega_0^2 h^2$$

Gerchikov, Guet, Ipatov, PRA 2002
GW, Ingold, Jalabert, Weinmann, PRB 2006

Mie frequency: $\omega_0 = \frac{\omega_{\mathrm{p}}}{\sqrt{1+2\epsilon_{\mathrm{m}}}}$ (visible, near-IR)

plasma frequency: $\omega_{\mathrm{p}} = \sqrt{\frac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}}$

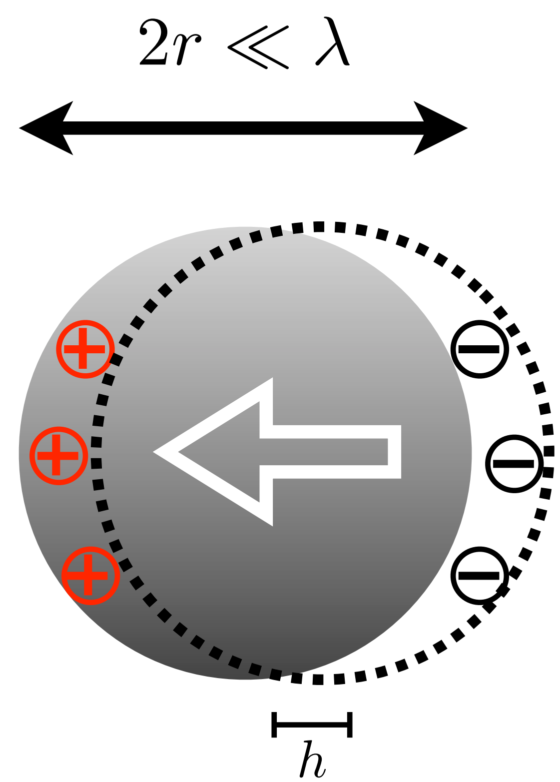
$M = N_{\mathrm{e}} m_{\mathrm{e}}$

Individual nanoparticle

Localized surface plasmon:

➡ dipolar collective excitation of the electronic center of mass

$$\mathbf{E}(t) = \mathbf{E}_0 \cos(\omega t)$$



(oscillating dipole moment)

$b \sim h + \mathrm{i}\,\Pi$

$$H_{\mathrm{LSP}} = \hbar\omega_0 \left(b^\dagger b + \frac{1}{2} \right)$$

Gerchikov, Guet, Ipatov, PRA 2002
GW, Ingold, Jalabert, Weinmann, PRB 2006

Mie frequency: $\omega_0 = \dfrac{\omega_{\mathrm{p}}}{\sqrt{1+2\epsilon_{\mathrm{m}}}}$ (visible, near-IR)

plasma frequency: $\omega_{\mathrm{p}} = \sqrt{\dfrac{4\pi n_{\mathrm{e}} e^2}{m_{\mathrm{e}}}}$

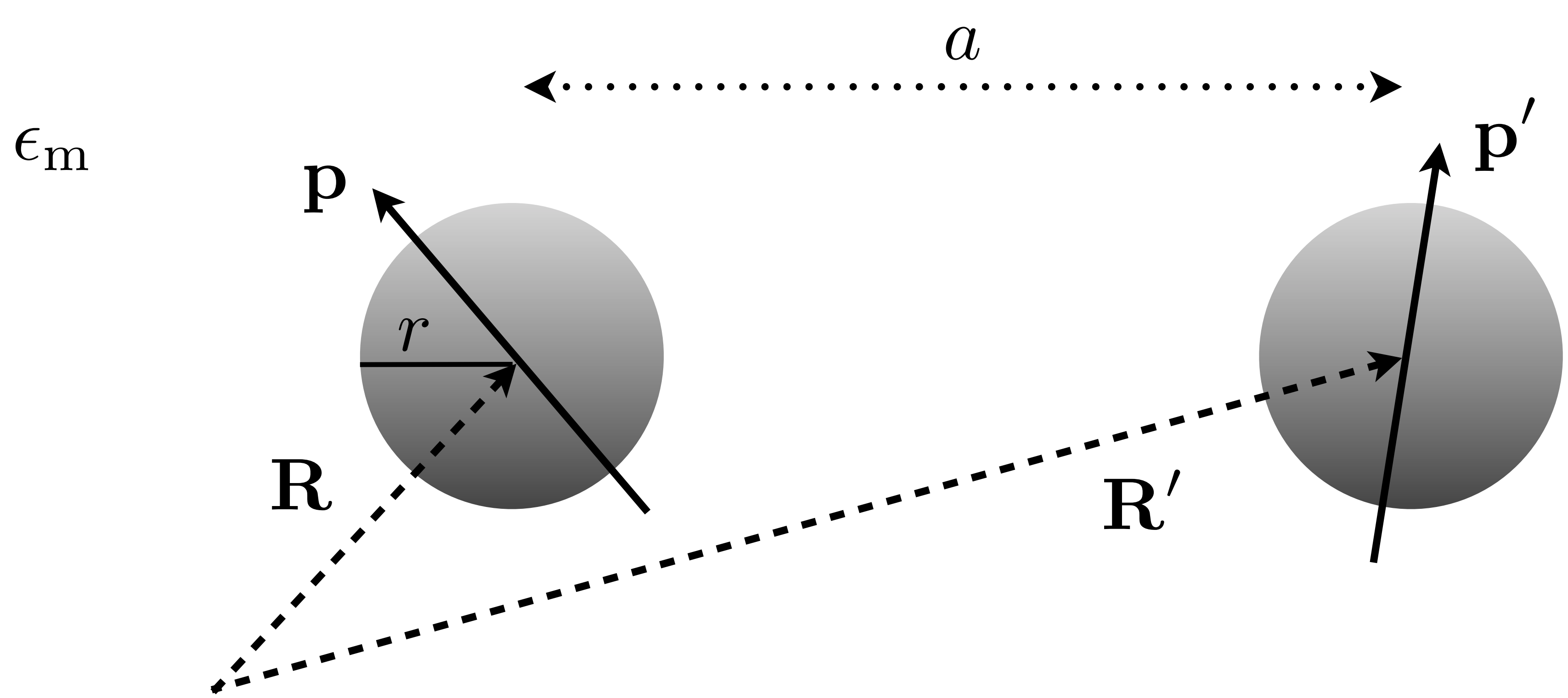
$M = N_{\mathrm{e}} m_{\mathrm{e}}$

Two nanoparticles

UNIVERSITÉ DE STRASBOURG

Dipole-dipole interaction:

➡ quasistatic approximation for point-like dipoles $(r \lesssim a/3 \ll \lambda)$



$$\mathcal{V} = \frac{\mathbf{p} \cdot \mathbf{p}' - 3(\mathbf{p} \cdot \mathbf{n})(\mathbf{p}' \cdot \mathbf{n})}{\epsilon_{\mathrm{m}} |\mathbf{R} - \mathbf{R}'|^3}$$

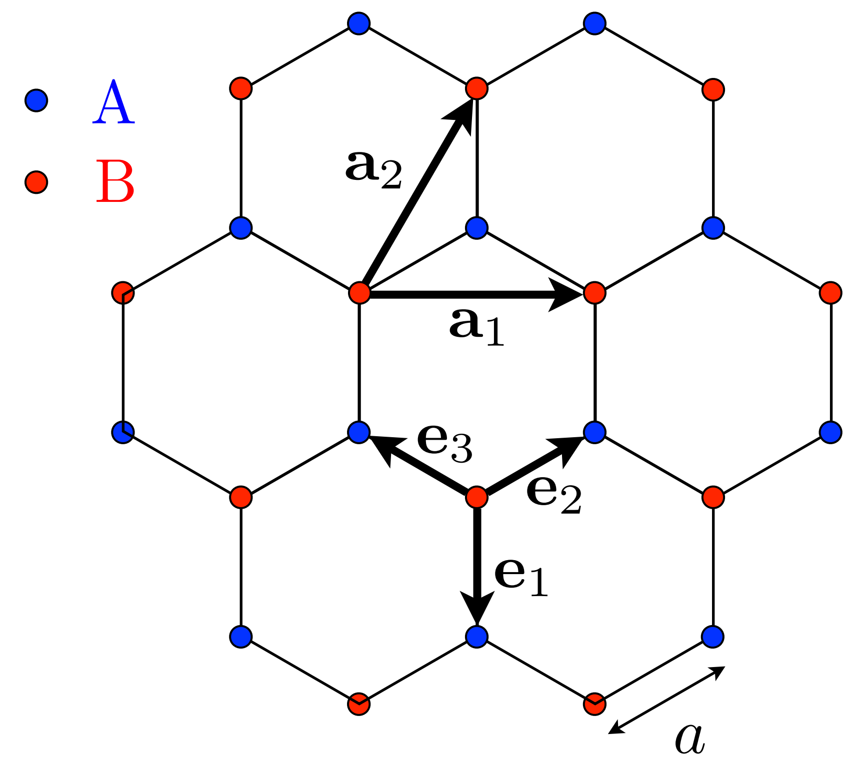
$$\mathbf{p} = -eN_{\mathrm{e}} h(\mathbf{R}) \hat{\mathbf{p}}$$

$$\mathbf{n} = \frac{\mathbf{R} - \mathbf{R}'}{|\mathbf{R} - \mathbf{R}'|}$$

Brongersma, Hartman, Atwater, PRB 2000
Park & Stroud, PRB 2004

Many nanoparticles

Honeycomb plasmonic lattice:



$$H_0 = \sum_{s=\mathrm{A,B}} \sum_{\mathbf{R}_s} \left[\frac{\Pi_s^2(\mathbf{R}_s)}{2M} + \frac{M}{2}\omega_0^2 h_s^2(\mathbf{R}_s) \right]$$

$$H_{\mathrm{int}} = \frac{(eN_\mathrm{e})^2}{\epsilon_\mathrm{m} a^3} \sum_{\mathbf{R}_\mathrm{B}} \sum_{j=1}^{3} \mathcal{C}_j h_\mathrm{B}(\mathbf{R}_\mathrm{B}) h_\mathrm{A}(\mathbf{R}_\mathrm{B} + \mathbf{e}_j)$$

(θ, φ): polarization of the dipoles

$$\mathcal{C}_j = 1 - 3\sin^2\theta\cos^2(\varphi - 2\pi[j-1]/3)$$

- nearest-neighbor interactions only

GW, Woollacott, Barnes, Hess, Mariani, PRL 2013

Analogy with electrons in graphene

UNIVERSITÉ DE STRASBOURG

Bosonic ladder operators:

$$H_{\mathrm{int}} = \hbar\Omega \sum_{\mathbf{R}_\mathrm{B}} \sum_{j=1}^{3} \mathcal{C}_j b^\dagger_{\mathbf{R}_\mathrm{B}} \left(a_{\mathbf{R}_\mathrm{B}+\mathbf{e}_j} + a^\dagger_{\mathbf{R}_\mathrm{B}+\mathbf{e}_j} \right) + \mathrm{H.c.}$$

$$a_\mathbf{R} = \sqrt{\frac{M\omega_0}{2\hbar}} h_\mathrm{A}(\mathbf{R}) + \frac{\mathrm{i}\Pi_\mathrm{A}(\mathbf{R})}{\sqrt{2\hbar M\omega_0}}$$

$$b_\mathbf{R} = \sqrt{\frac{M\omega_0}{2\hbar}} h_\mathrm{B}(\mathbf{R}) + \frac{\mathrm{i}\Pi_\mathrm{B}(\mathbf{R})}{\sqrt{2\hbar M\omega_0}}$$

$$\Omega = \omega_0 \left(\frac{r}{a}\right)^3 \frac{1+2\epsilon_\mathrm{m}}{6\epsilon_\mathrm{m}} \ll \omega_0$$

➡ cf. tight-binding Hamiltonian for electrons in graphene!

Graphene	Plasmonic graphene
fermions (electrons)	*bosons* (plasmons)
AB sublattices linked by kinetic process (hopping of electrons)	AB sublattices linked by *interactions* (dipole-dipole)
equal hopping matrix elements t	*tunable* couplings $\hbar\Omega\mathcal{C}_j$ (cf. strained graphene)
$\varnothing$	$H_0 = \hbar\omega_0 \sum_{\mathbf{R}_\mathrm{A}} a^\dagger_{\mathbf{R}_\mathrm{A}} a_{\mathbf{R}_\mathrm{A}} + \hbar\omega_0 \sum_{\mathbf{R}_\mathrm{B}} b^\dagger_{\mathbf{R}_\mathrm{B}} b_{\mathbf{R}_\mathrm{B}}$
$\varnothing$	anomalous term $\propto b^\dagger_{\mathbf{R}_\mathrm{B}} a^\dagger_{\mathbf{R}_\mathrm{B}+\mathbf{e}_j}$

Exact diagonalization

Université de Strasbourg

Starting Hamiltonian:

$$H = \hbar\omega_0 \sum_{\mathbf{q}} (a_{\mathbf{q}}^\dagger a_{\mathbf{q}} + b_{\mathbf{q}}^\dagger b_{\mathbf{q}}) + \hbar\Omega \sum_{\mathbf{q}} [f_{\mathbf{q}} b_{\mathbf{q}}^\dagger (a_{\mathbf{q}} + a_{-\mathbf{q}}^\dagger) + \mathrm{H.c.}]$$

$$f_{\mathbf{q}} = \sum_{j=1}^{3} \mathcal{C}_j \exp(\mathrm{i}\mathbf{q}\cdot\mathbf{e}_j)$$

Bogoliubov #1:

$$\alpha_{\mathbf{q}}^{\pm} = \frac{1}{\sqrt{2}} \left(\frac{f_{\mathbf{q}}}{|f_{\mathbf{q}}|} a_{\mathbf{q}} \pm b_{\mathbf{q}} \right)$$

$$\Rightarrow \quad H = \sum_{\tau=\pm} \sum_{\mathbf{q}} \left[(\hbar\omega_0 + \tau\hbar\Omega|f_{\mathbf{q}}|)\, {\alpha_{\mathbf{q}}^{\tau}}^\dagger \alpha_{\mathbf{q}}^{\tau} + \tau \frac{\hbar\Omega|f_{\mathbf{q}}|}{2} \left({\alpha_{\mathbf{q}}^{\tau}}^\dagger {\alpha_{-\mathbf{q}}^{\tau}}^\dagger + \mathrm{H.c.} \right) \right]$$

Bogoliubov #2:

$$\beta_{\mathbf{q}}^{\pm} = \cosh\vartheta_{\mathbf{q}}^{\pm} \alpha_{\mathbf{q}}^{\pm} - \sinh\vartheta_{\mathbf{q}}^{\pm} {\alpha_{-\mathbf{q}}^{\pm}}^\dagger$$

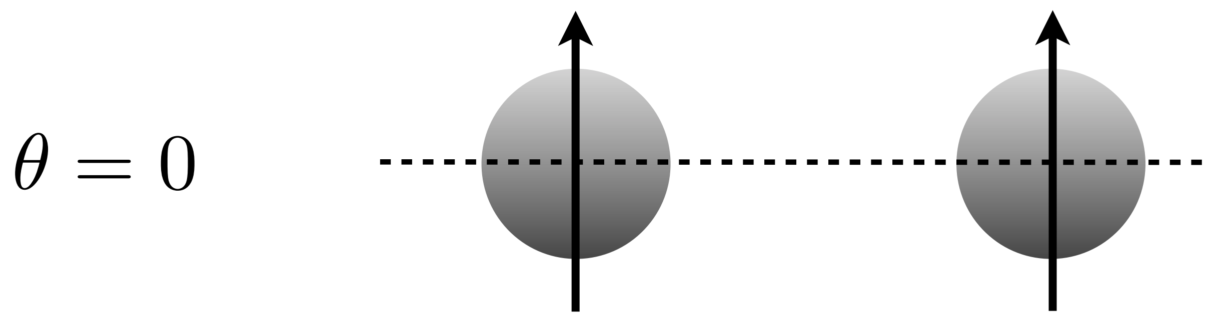
$$\cosh\vartheta_{\mathbf{q}}^{\pm} = 2^{-1/2}[(1 \pm \Omega|f_{\mathbf{q}}|/\omega_0)/(1 \pm 2\Omega|f_{\mathbf{q}}|/\omega_0)^{1/2} + 1]^{1/2}$$

$$\sinh\vartheta_{\mathbf{q}}^{\pm} = \mp 2^{-1/2}[(1 \pm \Omega|f_{\mathbf{q}}|/\omega_0)/(1 \pm 2\Omega|f_{\mathbf{q}}|/\omega_0)^{1/2} - 1]^{1/2}$$

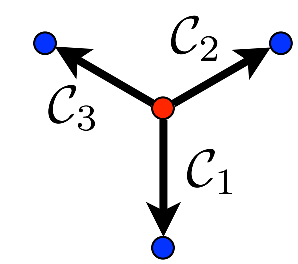
$$\Rightarrow \quad H = \sum_{\tau=\pm} \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}}^{\tau} {\beta_{\mathbf{q}}^{\tau}}^\dagger \beta_{\mathbf{q}}^{\tau}$$

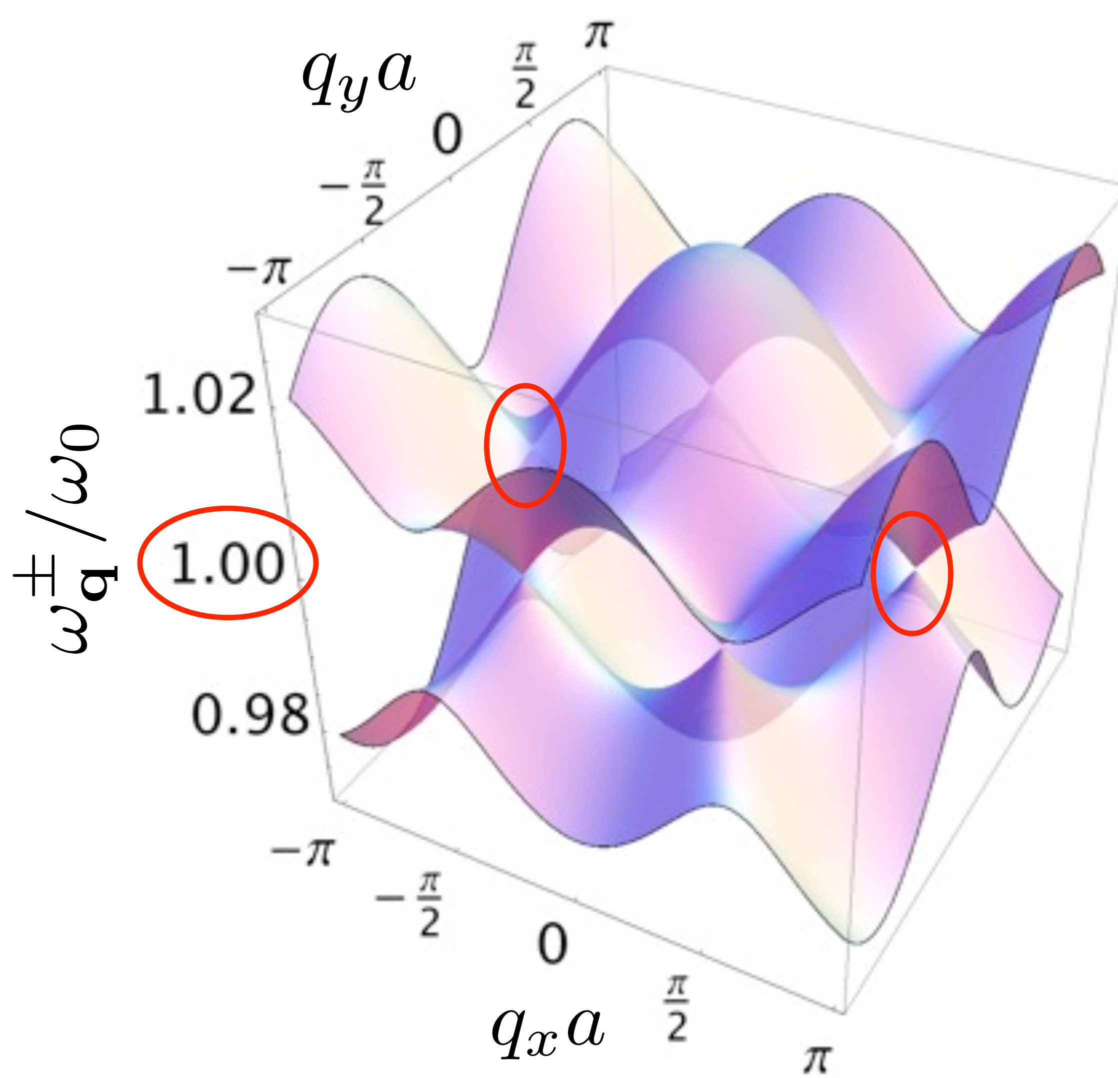
$$\omega_{\mathbf{q}}^{\pm} = \omega_0 \sqrt{1 \pm 2\frac{\Omega}{\omega_0}|f_{\mathbf{q}}|}$$

Plasmon dispersion



$\mathcal{C}_1 = \mathcal{C}_2 = \mathcal{C}_3 = 1$

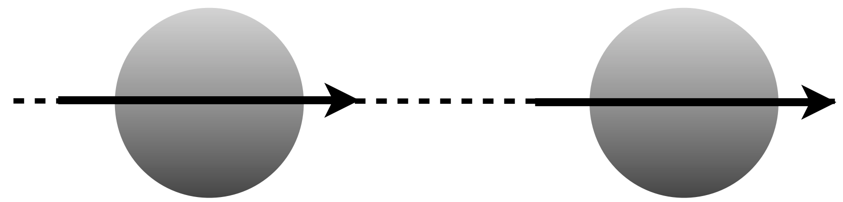




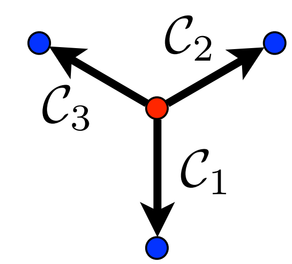
➡ two Dirac cones

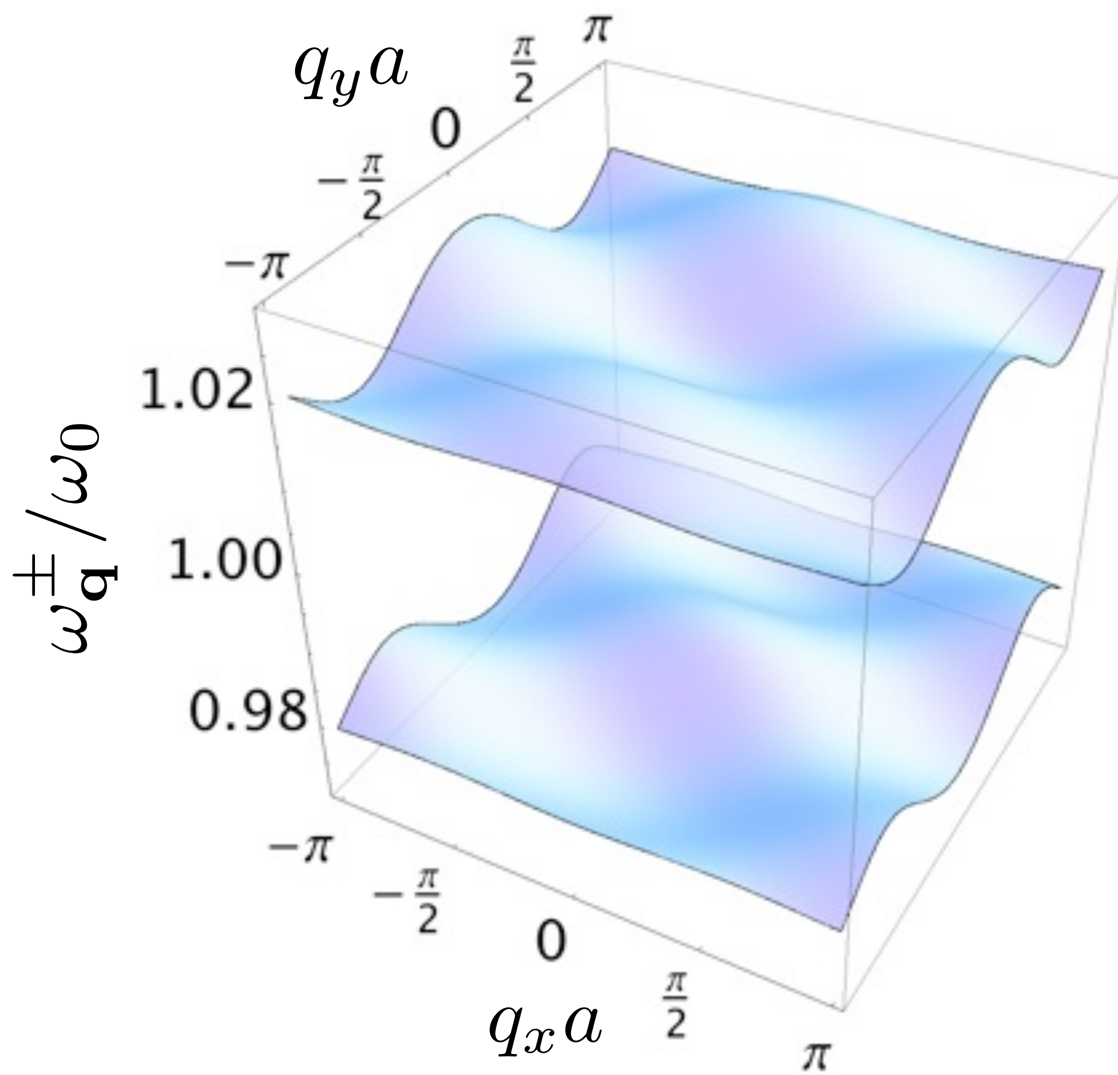
Plasmon dispersion

$$\theta = \frac{\pi}{2}$$



$$\mathcal{C}_1 = -2, \quad \mathcal{C}_2 = \mathcal{C}_3 = 1/4$$

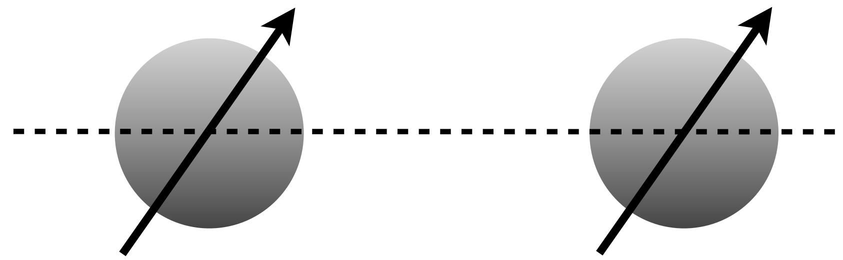




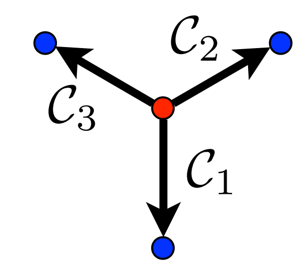
➡ gapped modes

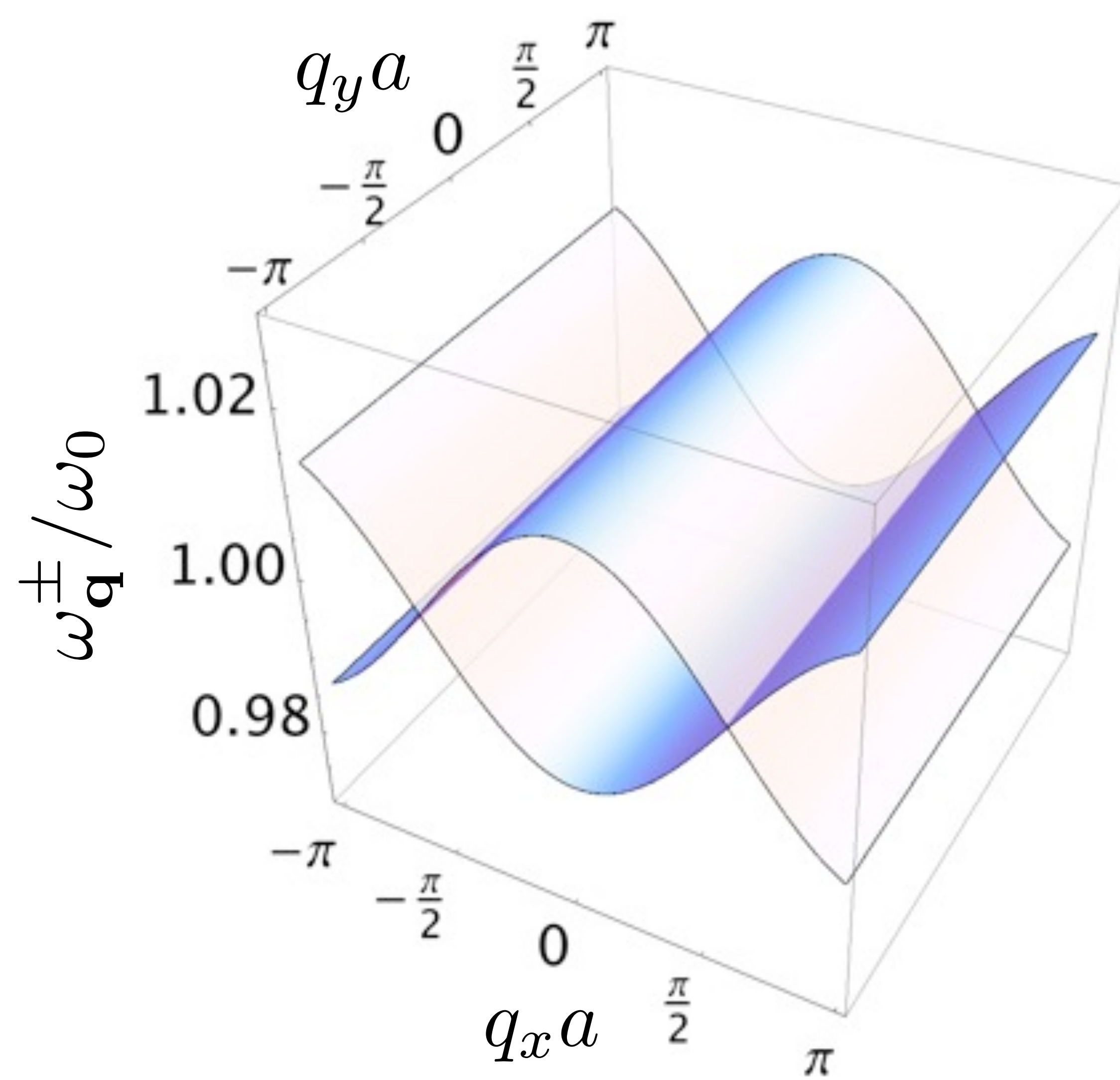
Plasmon dispersion

$$\theta = \arcsin\sqrt{\frac{1}{3}}$$



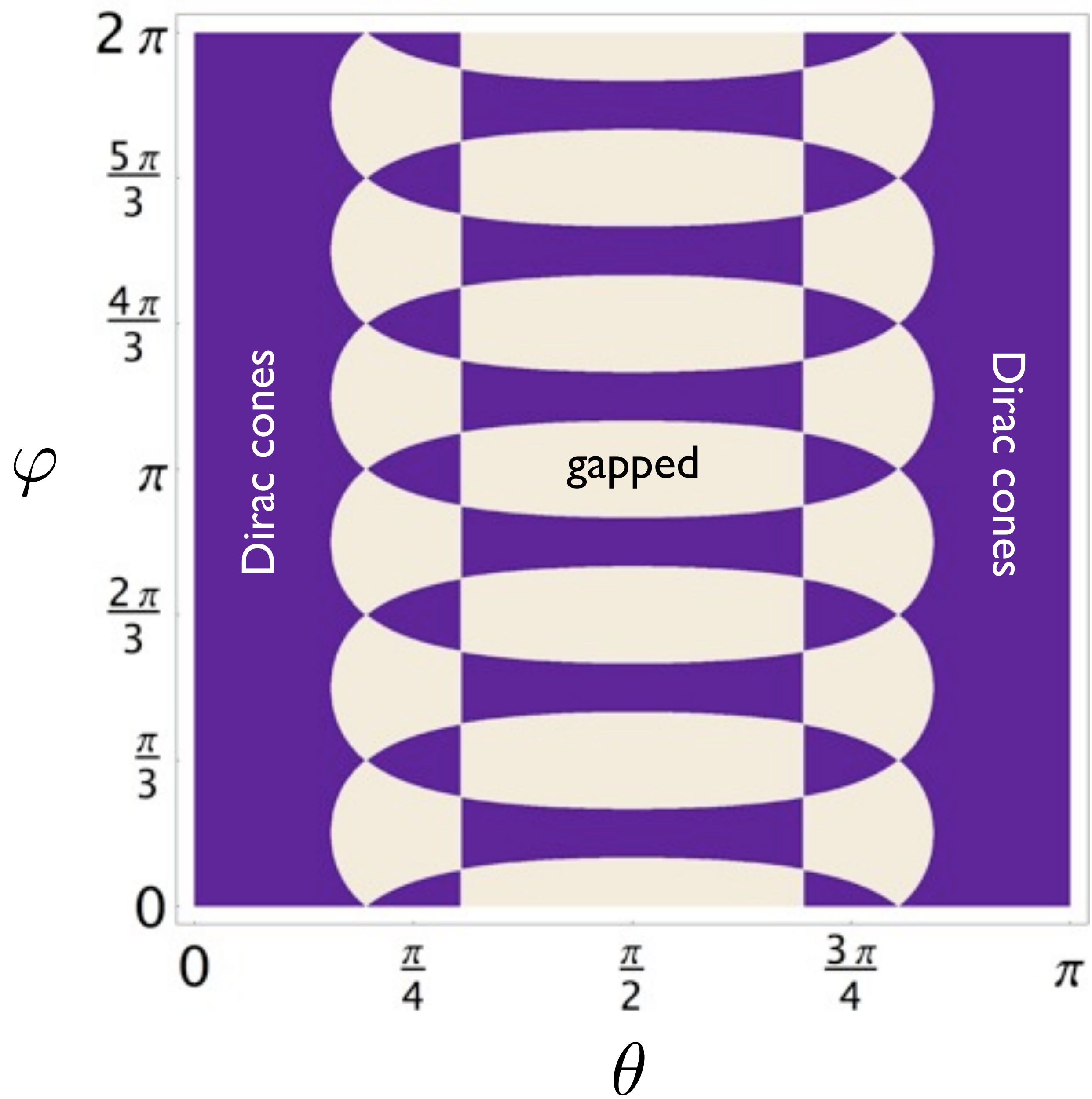
$\mathcal{C}_1 = 0, \quad \mathcal{C}_2 = \mathcal{C}_3 = 3/4$





➡ Dirac "lines"

Plasmon dispersion



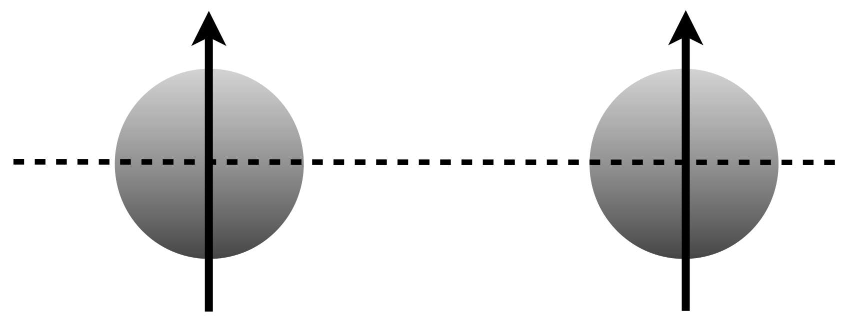
$$\omega_{\mathbf{q}}^{\pm} = \omega_0 \pm \Omega |f_{\mathbf{q}}|$$

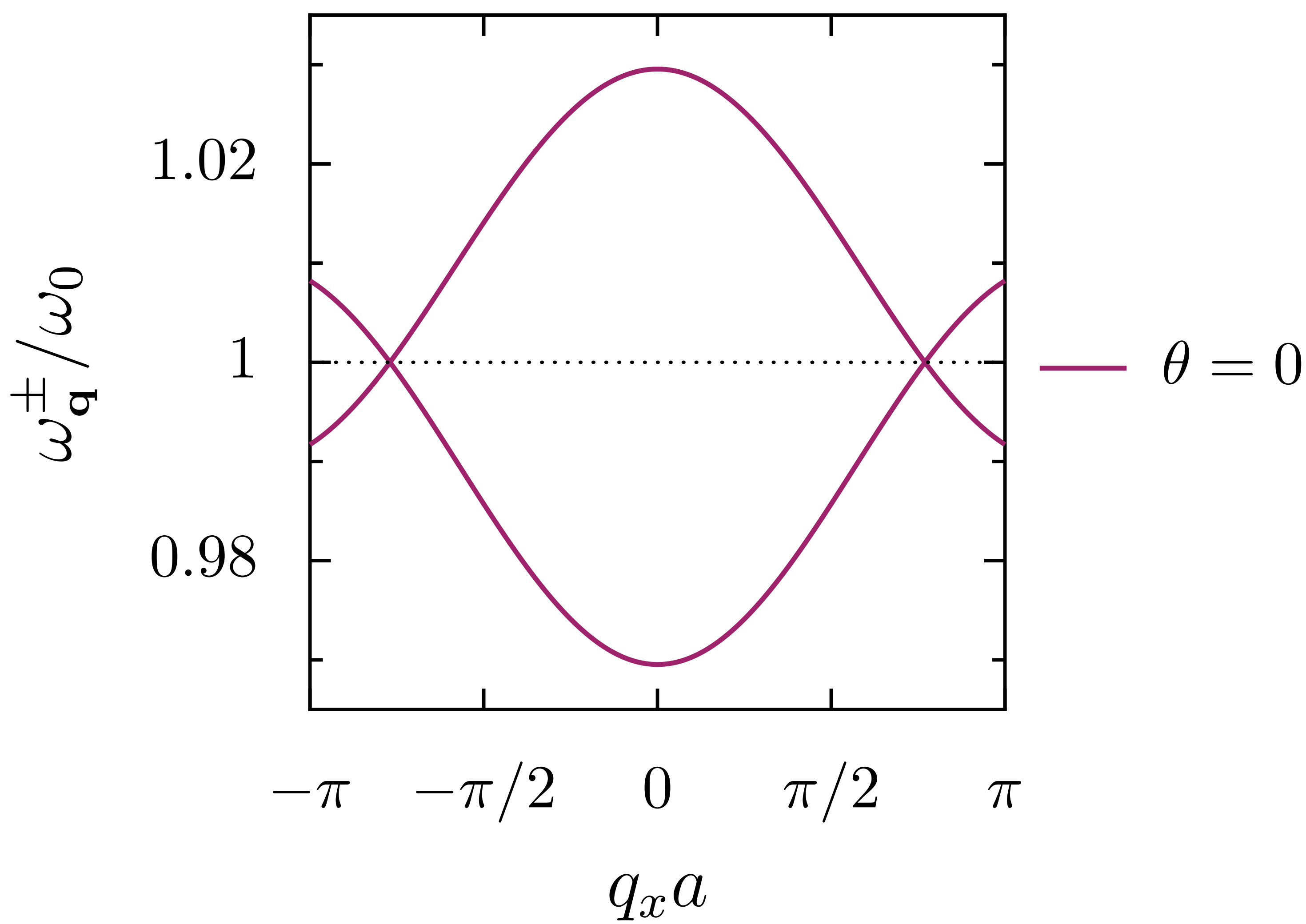
gapless modes: $|f_{\mathbf{q}}| = 0$

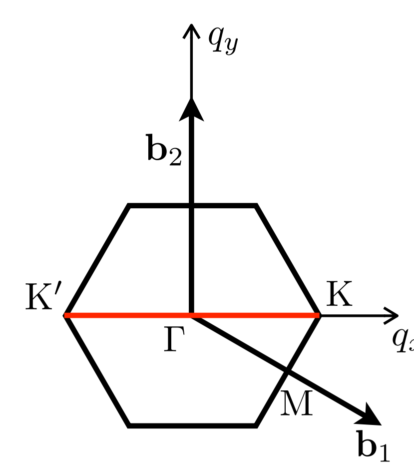
$$0 \leqslant \frac{(\mathcal{C}_2 + \mathcal{C}_3)^2 - \mathcal{C}_1^2}{4\mathcal{C}_2\mathcal{C}_3} \leqslant 1$$

➡ fully tunable spectrum (polarization)

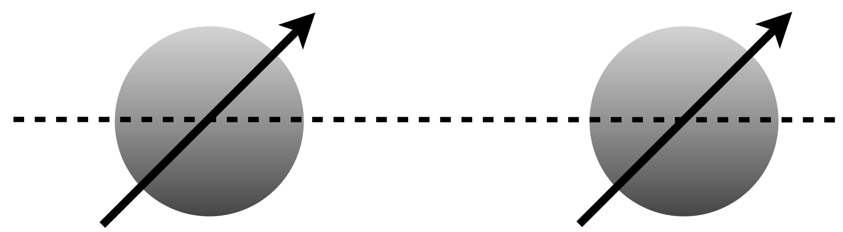
Plasmon dispersion

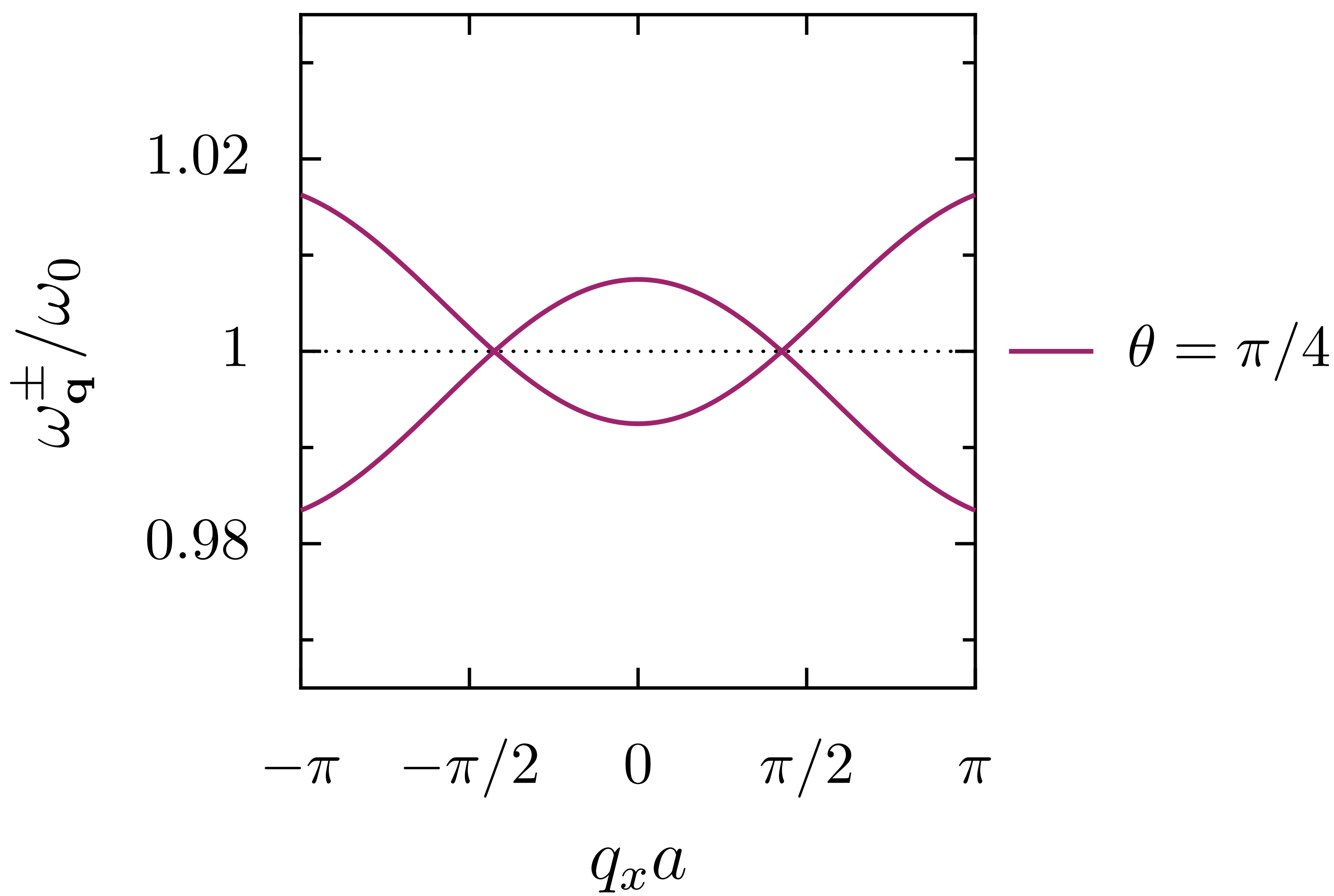


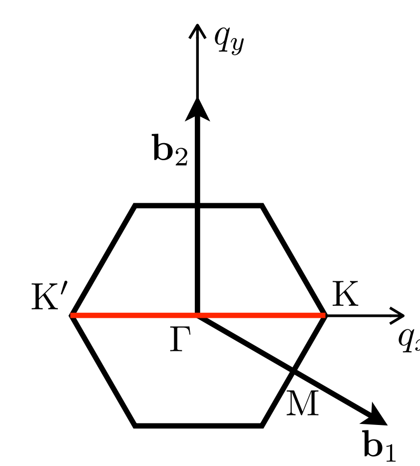




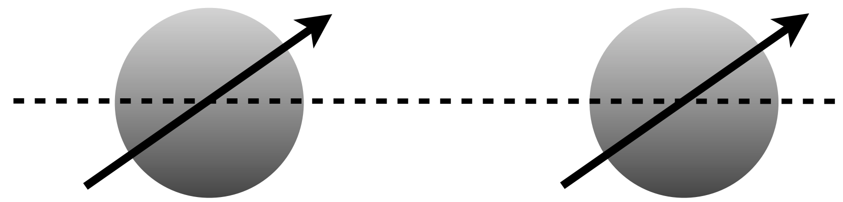
Plasmon dispersion



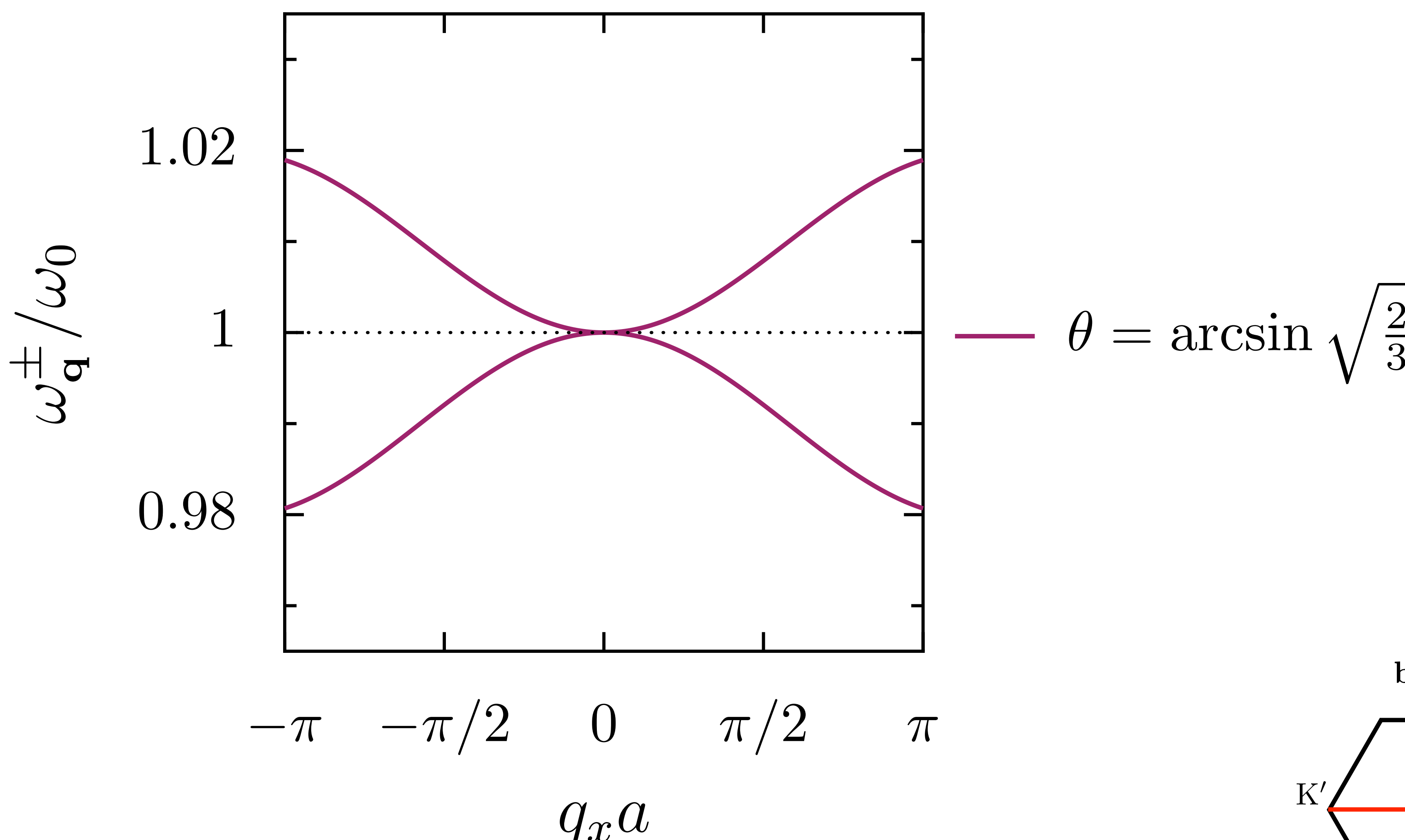


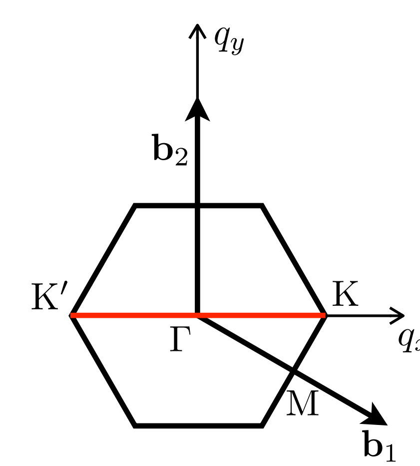


Plasmon dispersion

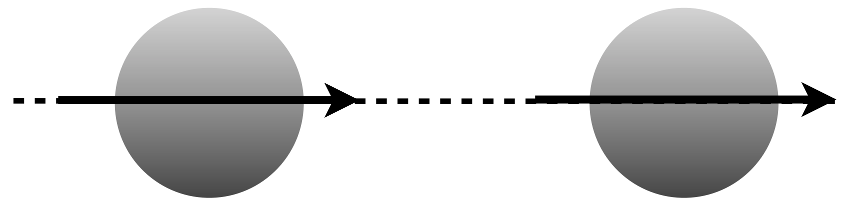


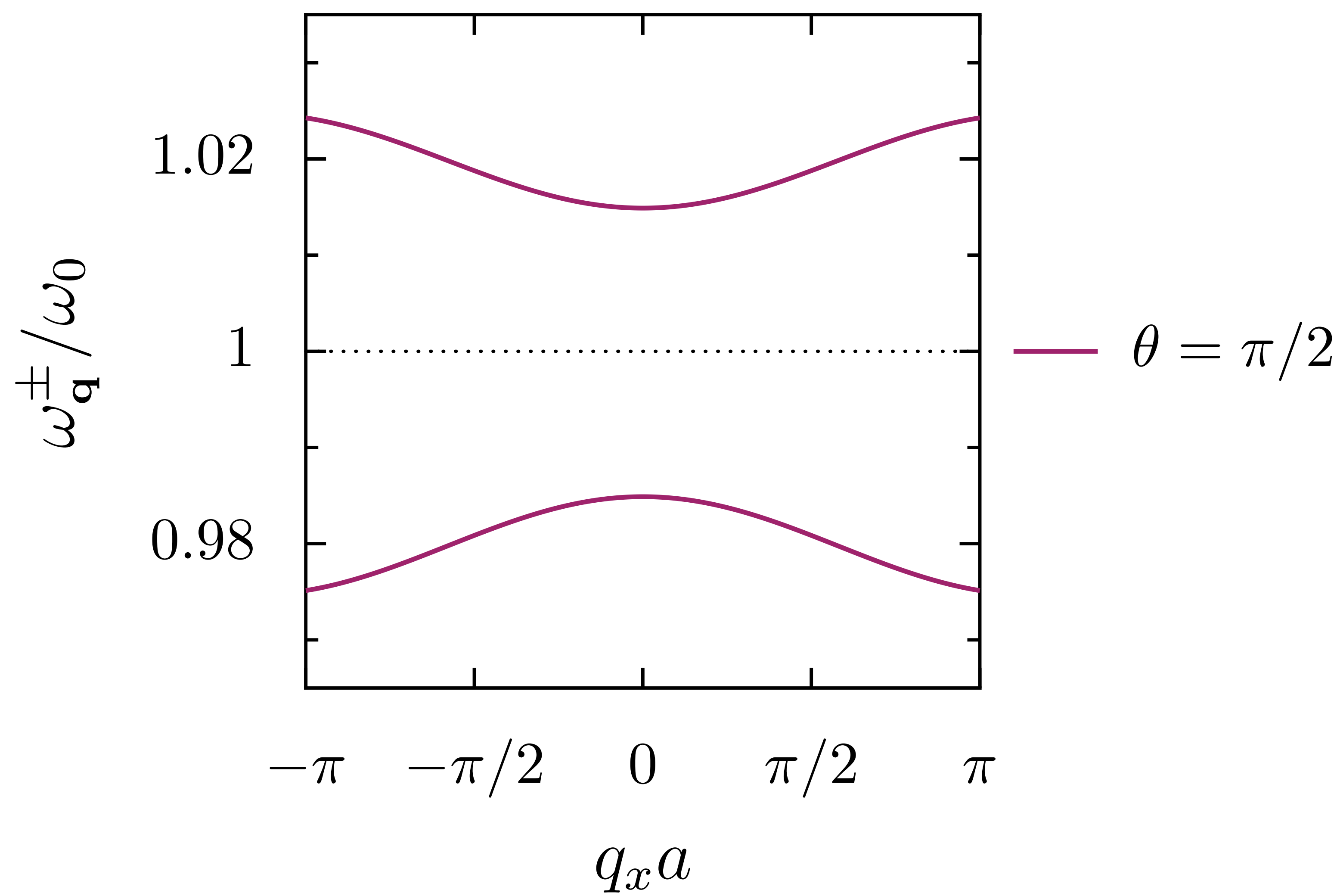
topological transition

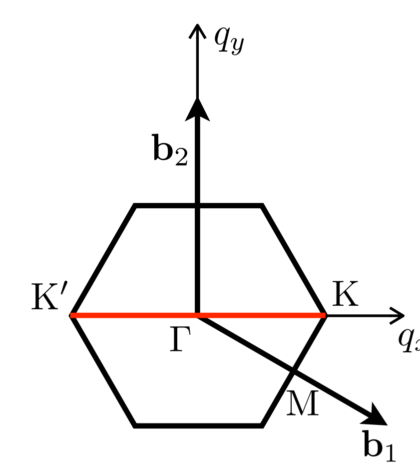




Plasmon dispersion

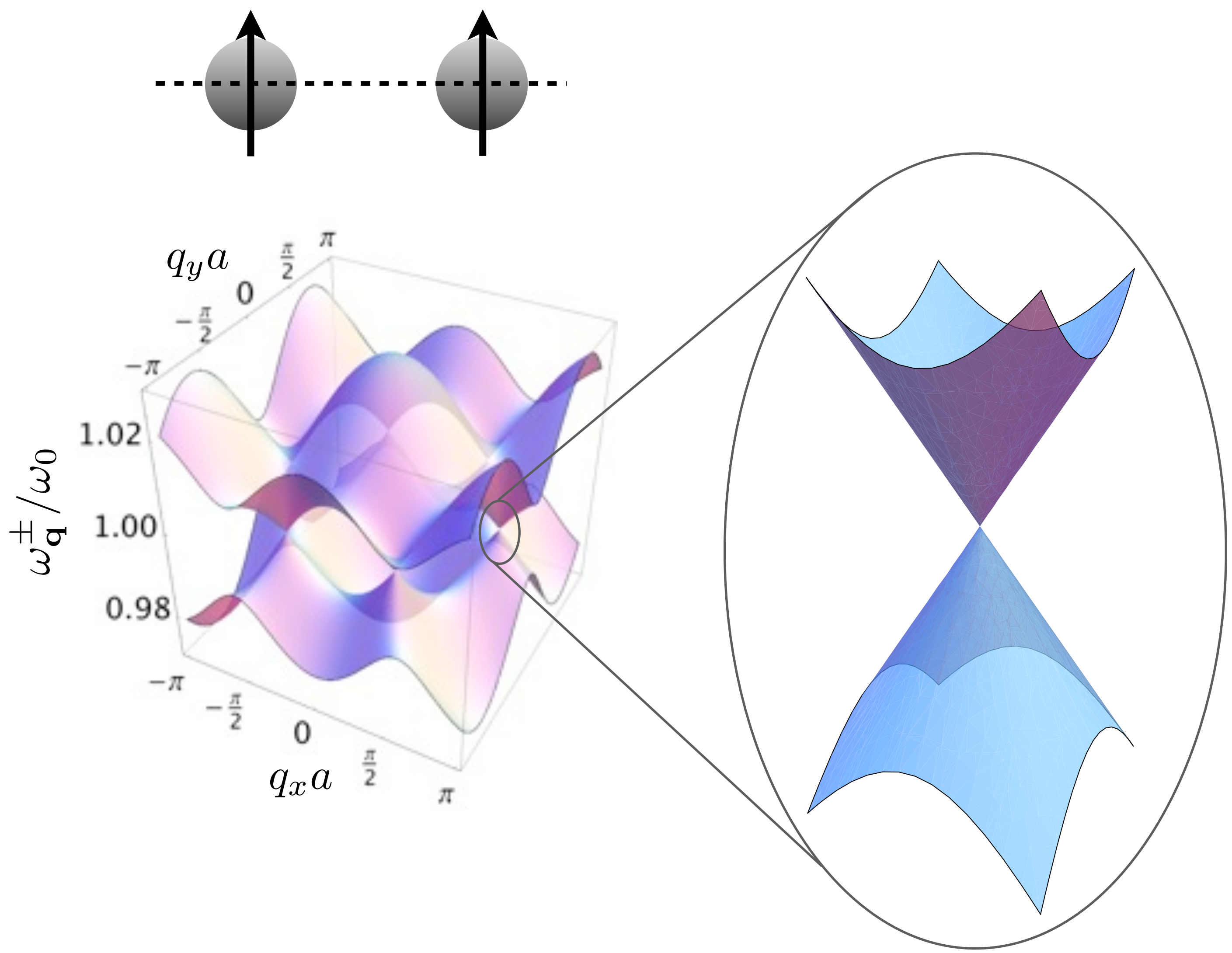






Dirac-like plasmons

UNIVERSITÉ DE STRASBOURG



Close to K point:

$$\omega^{\pm}_{\mathbf{k}} \simeq \omega_0 \pm v|\mathbf{k}|$$

group velocity: $v = 3\Omega a/2 \approx c/100$

$$\mathcal{H}^{\text{eff}}_{\mathbf{k}} = \hbar\omega_0 \mathbb{1} - \hbar v \tau_z \otimes \boldsymbol{\sigma} \cdot \mathbf{k}$$

spinor eigenstates:

$$\psi^{\pm}_{\mathbf{k},\mathrm{K}} = \frac{1}{\sqrt{2}}(1, \mathrm{e}^{\mp \mathrm{i}\xi_{\mathbf{k}}}, 0, 0)$$

chirality (helicity) $\boldsymbol{\sigma} \cdot \hat{\mathbf{k}} = \pm \mathbb{1}$

➡ Collective plasmons should show similar effects to electrons in graphene

- absence of backscattering
- Klein paradox
- Berry phase of π
- ...

Plasmon polaritons

UNIVERSITÉ DE STRASBOURG

How do plasmons couple to light in periodic arrays of nanoparticles?

PHYSICAL REVIEW VOLUME 103, NUMBER 5 SEPTEMBER 1, 1956

Atomic Theory of Electromagnetic Interactions in Dense Materials*

U. FANO
National Bureau of Standards, Washington, D. C.
(Received May 8, 1956)

PHYSICAL REVIEW VOLUME 112, NUMBER 5 DECEMBER 1, 1958

Theory of the Contribution of Excitons to the Complex Dielectric Constant of Crystals*†

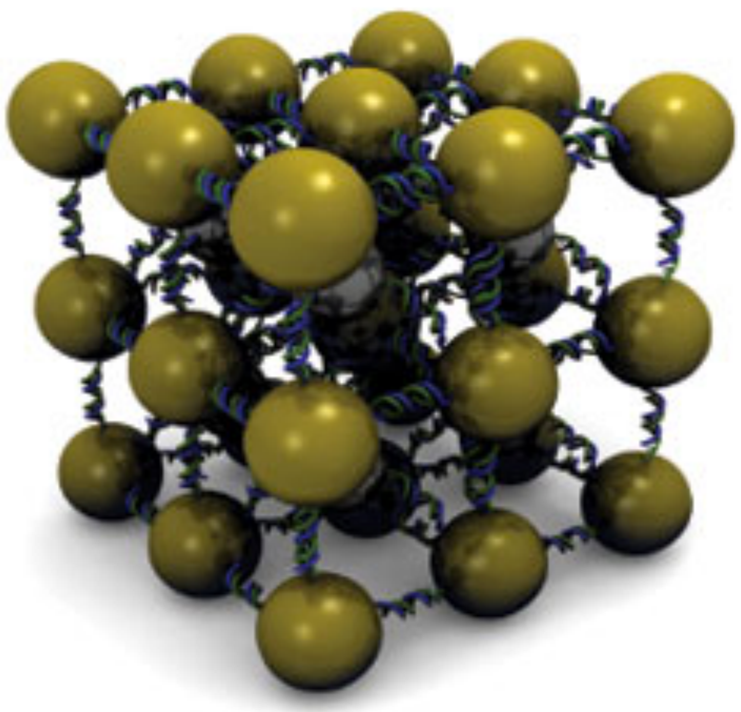
J. J. HOPFIELD‡
Physics Department, Cornell University, Ithaca, New York
(Received July 16, 1958)

➡ plasmon + photon = *plasmon polariton*

translational invariance: $\mathbf{k}_{\mathrm{photon}} = \mathbf{k}_{\mathrm{plasmon}}$

Simple cubic lattice

UNIVERSITÉ DE STRASBOURG



Tan *et al.*, Nature Nanotech. 2011

Conclusion

Plasmons in honeycomb lattices of metallic nanoparticles:

- massless Dirac-like *bosons*
- similar properties as electrons in graphene
- fully *tunable* spectrum

GW, C. Woollacott, W.L. Barnes, O. Hess, E. Mariani
Dirac-like plasmons in honeycomb lattices of metallic nanoparticles
Phys. Rev. Lett. **110**, 106801 (2013)

Plasmon polaritons in 3d (cubic) arrays:

- polaritonic band gap can be modified w/ light polarization
- tunable optical properties

GW, E. Mariani
Tunable plasmon polaritons in interacting arrays of metallic nanoparticles
unpublished