Kerr effect in layered nanostructures

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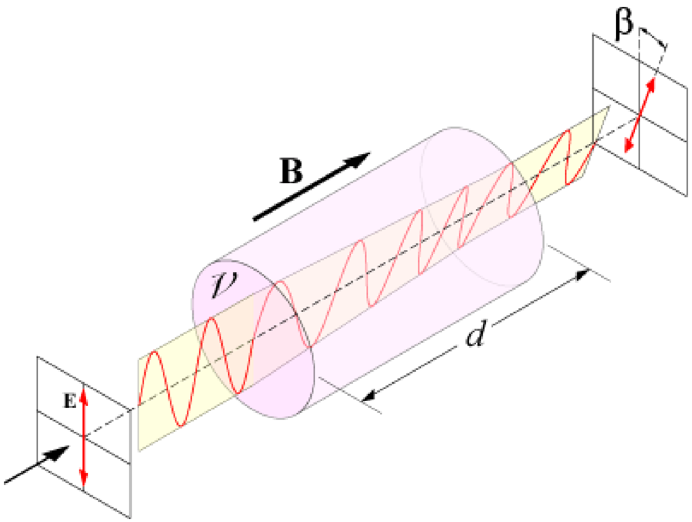
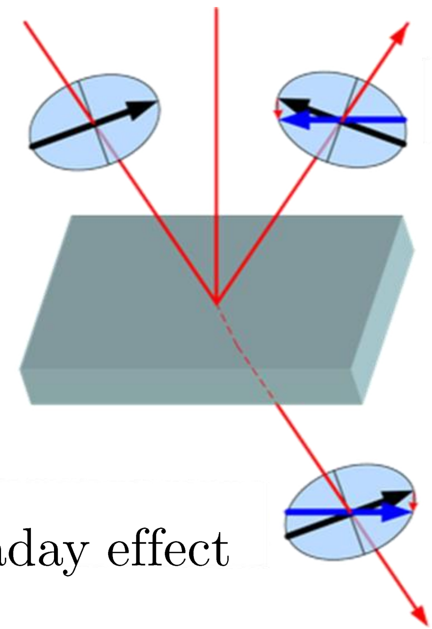
Faraday and Kerr effects

Magneto-optical phenomenon
Rotation of the plane of polarization



Faraday effect



Kerr effect

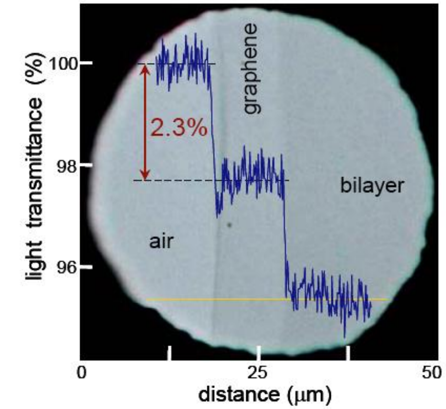
Faraday effect

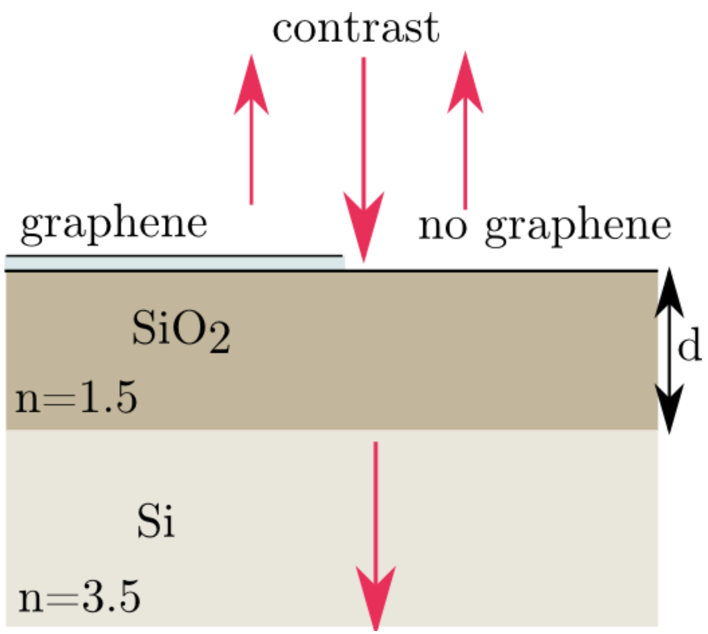
Broken time-reversal symmetry

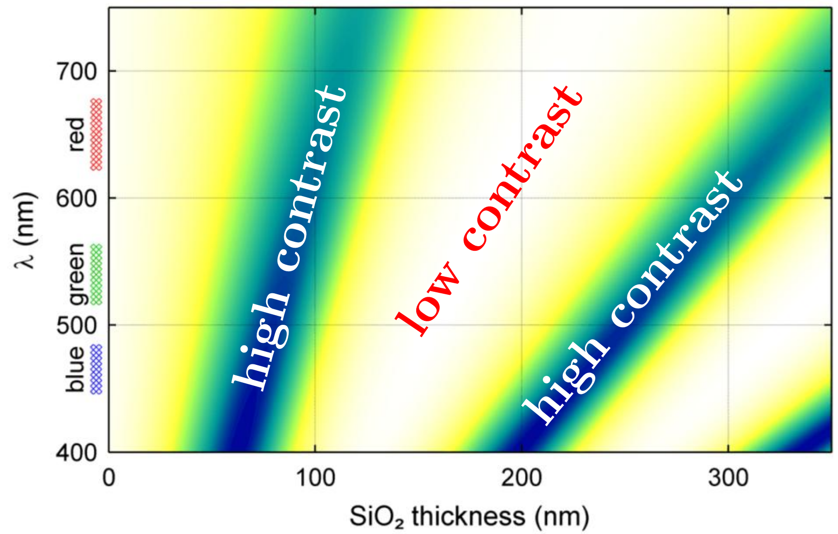
Motivation

Graphene visibility

- K. S. Novoselov ,et al., Science 306, 666 (2004)
- R.R. Nair et al., Science 320, 1308 (2008)
- P. Blake, et al., Appl. Phys. Lett. 91, 063124 (2007)



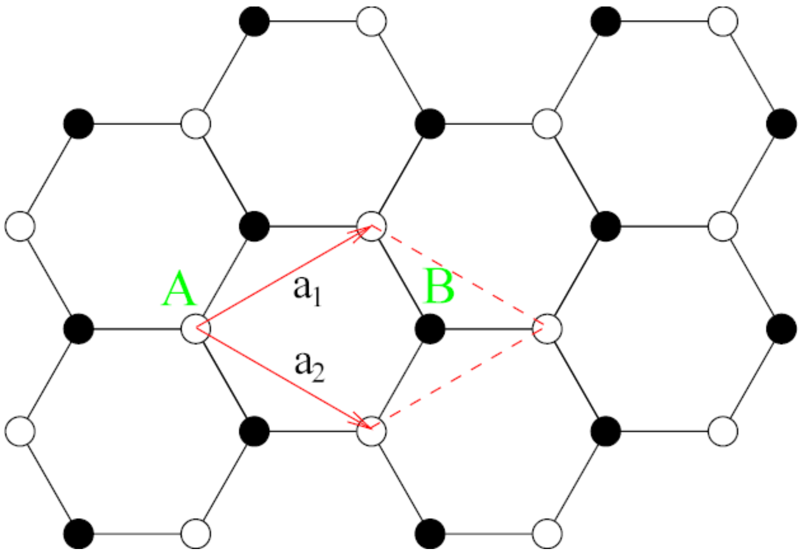


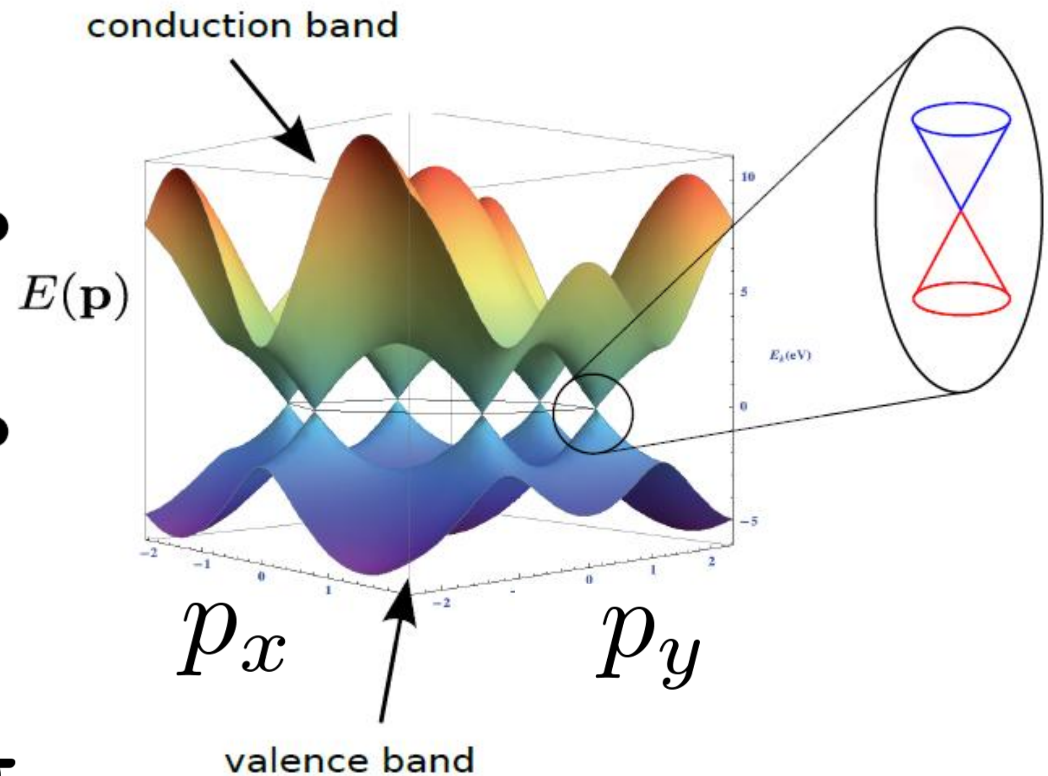


Kerr effect

Is Kerr-effect relevant in graphene nanostructures?
Calcualting Kerr-effect in layered nanostructures.

Graphene





Hamiltonian: $H = v\mathbf{p}\boldsymbol{\sigma}$

Complex conductivity:

$$\sigma_{\alpha\beta}(\omega) = \frac{e^2}{4\hbar}\delta_{\alpha\beta}\left(4\mu\delta(\hbar\omega) + \Theta(\hbar\omega - 2\mu) + i\frac{4\mu}{\pi\hbar\omega} - \frac{i}{\pi}\log\frac{\hbar\omega+2\mu}{\hbar\omega-2\mu}\right)$$

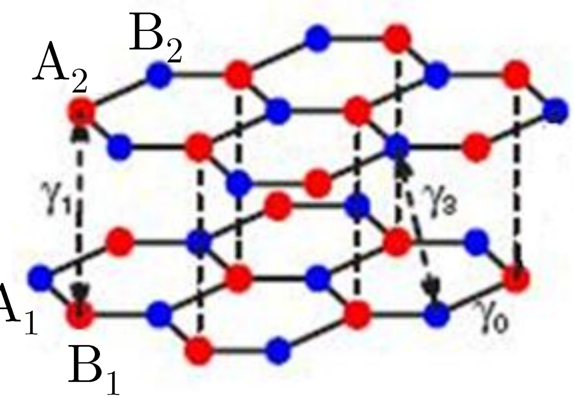
Diagonal $\rightarrow$ No Kerr

Chemical potential

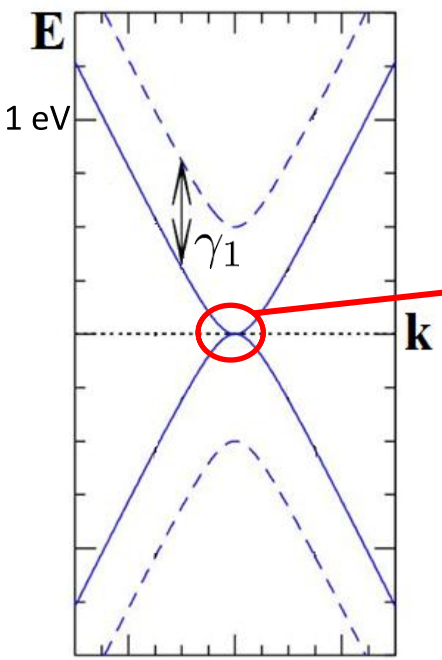
Bilayer graphene

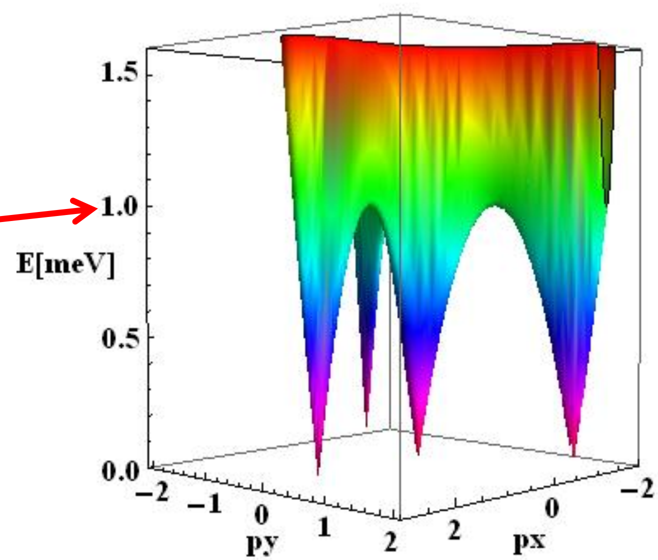
Hamiltonian:

$$\hat{H} = \begin{pmatrix} 0 & \beta v p_+ & 0 & v p_- \\ \beta v p_- & 0 & v p_+ & 0 \\ 0 & v p_- & 0 & \gamma_1 \\ v p_+ & 0 & \gamma_1 & 0 \end{pmatrix} \begin{pmatrix} A_1 \\ B_2 \\ A_2 \\ B_1 \end{pmatrix}$$



Dispersion relation:





Trigonal warping

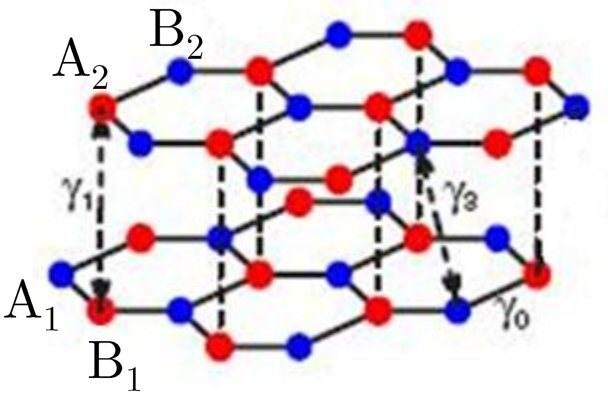
- $v = 3.7 \cdot 10^{-3} c$
- $v = \frac{\sqrt{3} a \gamma_0}{2\hbar}$
- $\gamma_1 = 0.39 eV$
- $p_\pm = p_x \pm i p_y$
- $\beta = \frac{\gamma_3}{\gamma_0} \approx 0.1$

Neglect: β=0

Broken symmetry in bilayer graphene

Order parameter

$$\hat{H} = \xi \begin{pmatrix} \Delta_{\xi s} & 0 & 0 & v_F p_- \\ 0 & -\Delta_{\xi s} & v_F p_+ & 0 \\ 0 & v_F p_- & 0 & \xi\gamma_1 \\ v_F p_+ & 0 & \xi\gamma_1 & 0 \end{pmatrix} \begin{pmatrix} A_1 \\ B_2 \\ A_2 \\ B_1 \end{pmatrix}$$



Valley quantumnumber ± 1

$$\Delta_{\xi s} = \Delta_{\mathrm{QVH}} + s\Delta_{\mathrm{LAF}} + \xi\Delta_{\mathrm{QAH}} + s\xi\Delta_{\mathrm{QSH}}$$

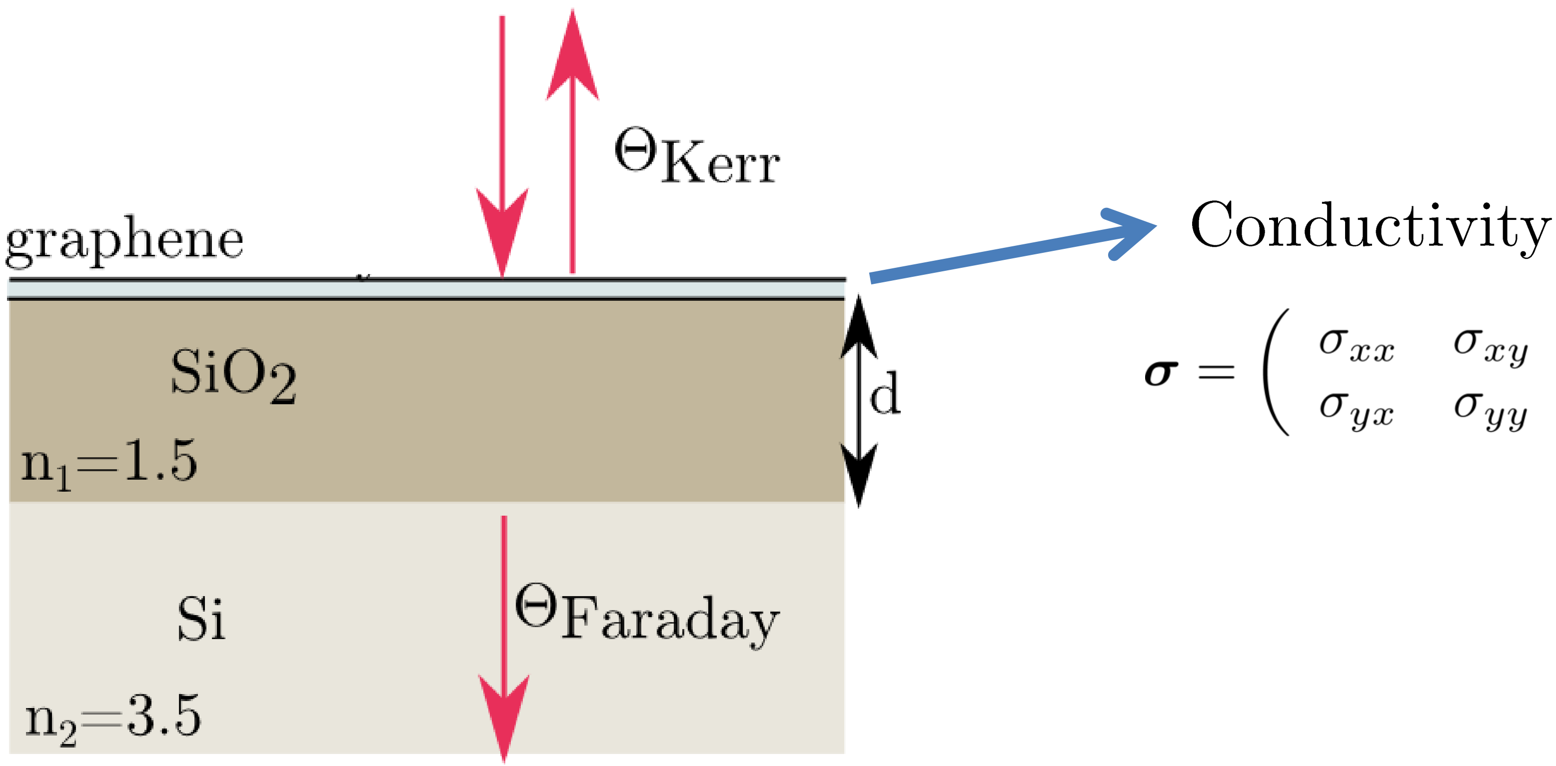
Possible gapped states in bilayer graphene at neutral point:

Ordered state	Order parameter
QVH	$\langle \psi^\dagger_{A1Ks}\psi_{A1Ks} + \psi^\dagger_{A1K's}\psi_{A1K's} - \psi^\dagger_{B2Ks}\psi_{B2Ks} - \psi^\dagger_{B2K's}\psi_{B2K's}\rangle$
LAF	$\langle \psi^\dagger_{A1Ks}s\psi_{A1Ks} + \psi^\dagger_{A1K's}s\psi_{A1K's} - \psi^\dagger_{B2Ks}s\psi_{B2Ks} - \psi^\dagger_{B2K's}s\psi_{B2K's}\rangle$
QAH	$\langle \psi^\dagger_{A1Ks}\psi_{A1Ks} - \psi^\dagger_{A1K's}\psi_{A1K's} - \psi^\dagger_{B2Ks}\psi_{B2Ks} + \psi^\dagger_{B2K's}\psi_{B2K's}\rangle$
QSH	$\langle \psi^\dagger_{A1Ks}s\psi_{A1Ks} - \psi^\dagger_{A1K's}s\psi_{A1K's} - \psi^\dagger_{B2Ks}s\psi_{B2Ks} + \psi^\dagger_{B2K's}s\psi_{B2K's}\rangle$

- E. V. Gorbar ,et al., PRB 86, 075414 (2012)
- R. Nandkishore, et al., PRL 107, 097402 (2011)

Transfer matrix method I.

Example:

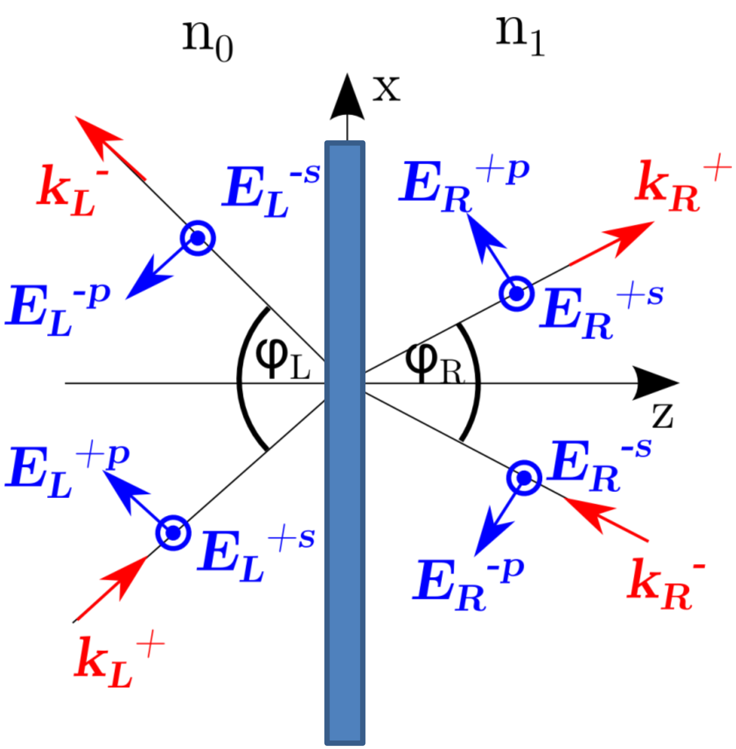


$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}$$

Structure of transfer matrix:

$$\mathbf{M_{jump}}\left(n_1, n_2\right)\mathbf{M_{propagate}}\left(n_1, d\right)\mathbf{M_{jump}}\left(n_0, n_1, \boldsymbol{\sigma}\right)$$

Transfer matrix method II.



Graphene as a boundary

Boundary condition:

$$\mathrm{rot}\mathbf{H} = \boldsymbol{\sigma}\mathbf{E} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathrm{rot}\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\begin{pmatrix} E_R^{+s} \\ E_R^{+p} \\ E_R^{-s} \\ E_R^{-p} \end{pmatrix} = \mathbf{M_{jump}} \begin{pmatrix} E_L^{+s} \\ E_L^{+p} \\ E_L^{-s} \\ E_L^{-p} \end{pmatrix}$$

$\sigma_{xy} = 0$ → No coupling between p and s mode
No Kerr rotation

Transfer matrix method III.

$$\mathbf{M_{total}} = \mathbf{M_{jump}}(n_1, n_2)\,\mathbf{M_{propagate}}(n_1, d)\,\mathbf{M_{jump}}(n_0, n_1, \boldsymbol{\sigma})$$

transmission

$$\mathbf{t} = \begin{pmatrix} t_{ss} & t_{sp} \\ t_{ps} & t_{pp} \end{pmatrix}$$

Faraday effect

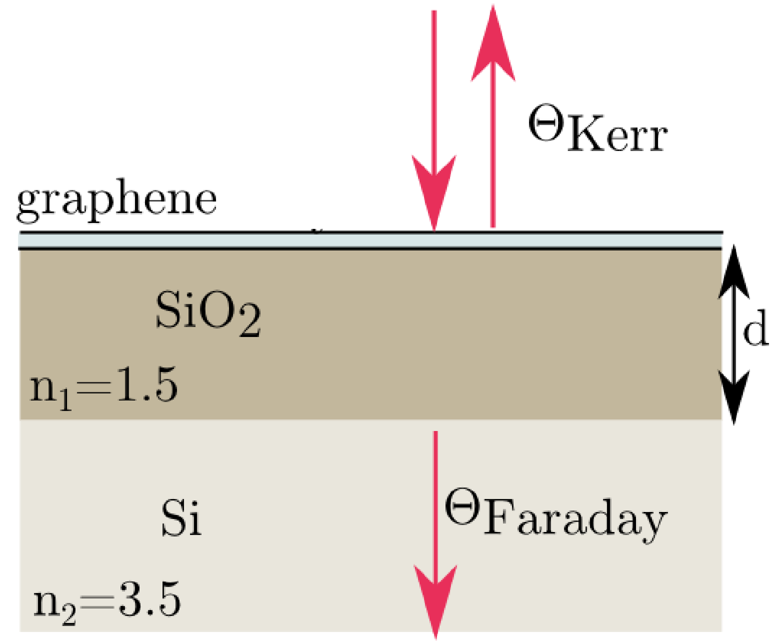
$$\Theta_{Faraday} = \mathrm{Re}\left[\frac{t_{sp}}{t_{pp}}\right]$$

reflection

$$\mathbf{r} = \begin{pmatrix} r_{ss} & r_{sp} \\ r_{ps} & r_{pp} \end{pmatrix}$$

Kerr effect

$$\Theta_{Kerr} = \mathrm{Re}\left[\frac{r_{sp}}{r_{pp}}\right]$$



Need σ_{xy} !!

Kerr angle

Kerr angle at perpendicular incidence

$k = \frac{n_1}{\omega c}$

$$\Theta_{Kerr} = \frac{\left(n_1+e^{2ikd}(n_1-n_2)+n_2\right)^2}{(e^{2ikd}(1+n_1)(n_1-n_2)-(n_1-1)(n_1+n_2))(e^{2ikd}(1-n_1)(n_1-n_2)+(n_1+1)(n_1+n_2))}\mathrm{Re}\left[\frac{\sigma_{xy}}{\epsilon_0 c}\right]$$

1) $d = \frac{n\pi}{k} \qquad n \in \mathbb{N}$

$$\Theta_{Kerr} = \frac{2}{1-n_2^2}\mathrm{Re}\left[\frac{\sigma_{xy}}{\epsilon_0 c}\right]$$

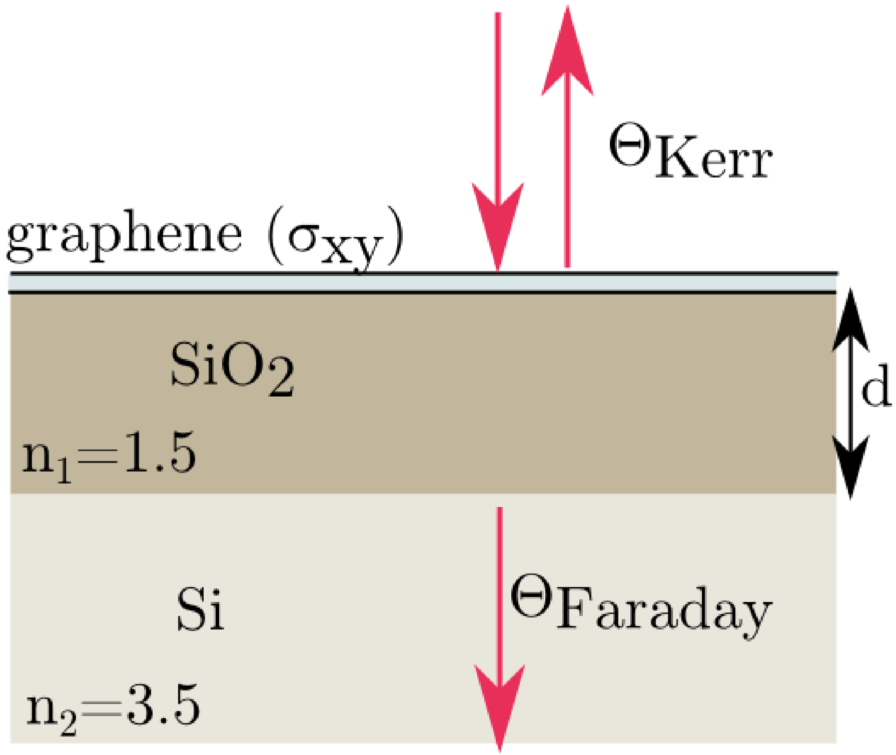
Large Kerr rotation if: $n_2 = 1$

Free-standing graphene

2) $d = \frac{\pi}{2k} + \frac{n\pi}{k} \qquad n \in \mathbb{N}$

$$\Theta_{Kerr} = \frac{2n_2^2}{n_2^2-n_1^4}\mathrm{Re}\left[\frac{\sigma_{xy}}{\epsilon_0 c}\right]$$

Large Kerr rotation if: $n_1 = \sqrt{n_2}$



Optical Hall conductivity

Current-current correlation function

$$\Pi_{xy}(i\omega) = \frac{1}{V\beta} \sum_{\mathbf{k},n} \mathrm{Tr}\left[\frac{\partial H}{\partial p_x} G(\mathbf{k}, i\omega_n + i\omega) \frac{\partial H}{\partial p_y} G(\mathbf{k}, i\omega_n) \right],$$

Green's function

$$G(\mathbf{k}, i\omega_n) = \frac{1}{i\omega_n - H}$$

Matsubara frequency

Complex conductivity

$$\sigma_{xy}(\omega) = \frac{ie^2}{\omega} \Pi_{xy}(i\omega \to \omega + i\eta)$$

Zero approximation of self-energy

Numerically versatile method using projectors

Hall conductivity of bilayer graphene

QAH state in bilayer graphene:

$$\Delta_{\xi s} = \xi \Delta_{QAH}$$

$$\hat{H} = \xi \begin{pmatrix} \Delta_{\xi s} & 0 & 0 & v_F p_- \\ 0 & -\Delta_{\xi s} & v_F p_+ & 0 \\ 0 & v_F p_- & 0 & \xi\gamma_1 \\ v_F p_+ & 0 & \xi\gamma_1 & 0 \end{pmatrix}$$

Eigenvalues:

$$E_{1,2}^2 = v_F^2 p^2 + \frac{\Delta_{\xi s}^2 + \gamma_1^2}{2} \pm \sqrt{\frac{(\gamma_1^2 - \Delta_{\xi s}^2)^2}{4} + (\gamma_1^2 + \Delta_{\xi s}^2) v_F^2 p^2},$$

Current-current correlation function

$$\Pi_{xy}(i\omega) = \sum_{\xi=\pm, s=\pm} \frac{i\xi\Delta_{\xi s}}{2\pi} \int_0^\infty p\,dp \left\{ \frac{\left(\gamma_1^2 - \Delta_{\xi s}^2\right)(E_1+E_2)\left(\gamma_1^2 - p^2 - E_1E_2\right)}{E_1E_2(E_1^2 - E_2^2)^2} \left(\frac{1}{i\omega + E_1 + E_2} + \frac{1}{i\omega - E_1 - E_2}\right) \right.$$
$$\left. + \frac{4p^2\gamma_1^2}{(E_1^2 - E_2^2)^2} \left[\frac{1}{E_1}\left(\frac{1}{i\omega + 2E_1} + \frac{1}{i\omega - 2E_1}\right) + \frac{1}{E_2}\left(\frac{1}{i\omega + 2E_2} + \frac{1}{i\omega - 2E_2}\right)\right] \right\}.$$

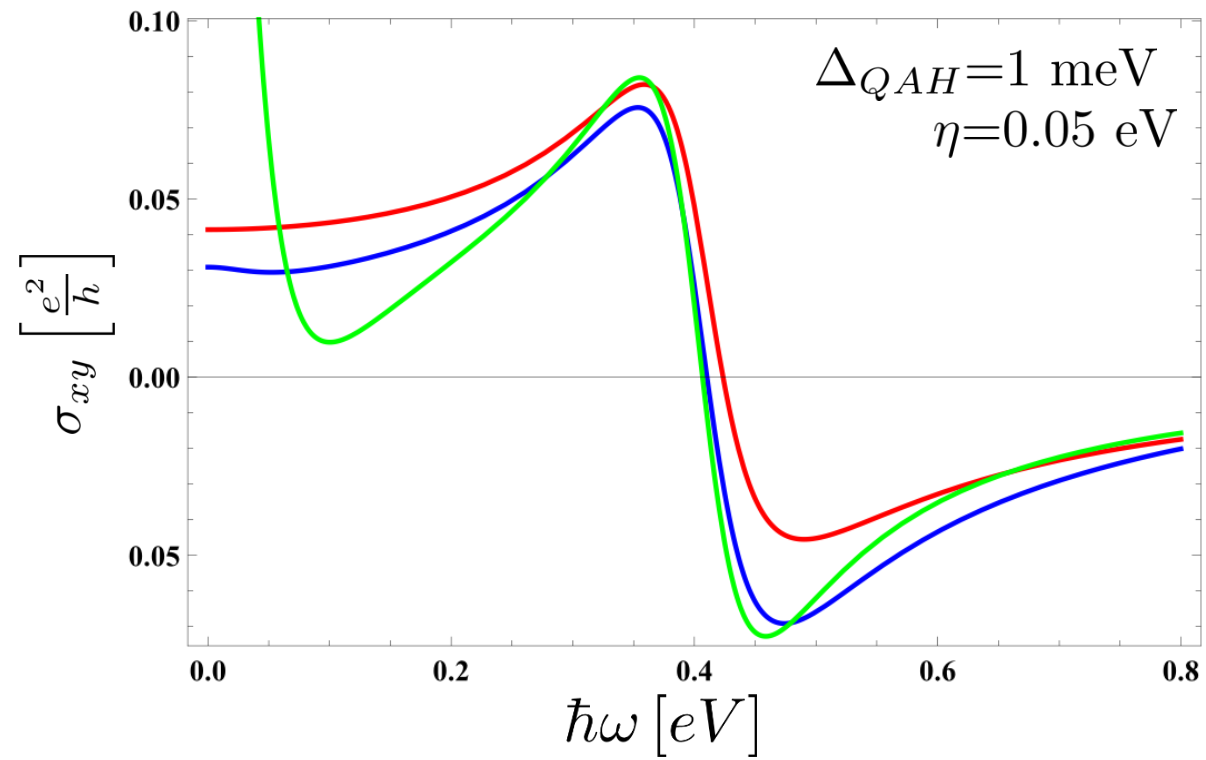
Complex Hall conductivity

$$\sigma_{xy}(\omega) = \frac{ie^2}{\omega} \Pi_{xy}(i\omega \to \omega + i\eta)$$

Hall conductivity of bilayer graphene

$$\hat{H} = \xi \begin{pmatrix} \Delta_{\xi s} & 0 & 0 & v_F p_- \\ 0 & -\Delta_{\xi s} & v_F p_+ & 0 \\ 0 & v_F p_- & 0 & \xi\gamma_1 \\ v_F p_+ & 0 & \xi\gamma_1 & 0 \end{pmatrix}$$

$$\Delta_{\xi s} = \xi \Delta_{QAH}$$



IR spectral range

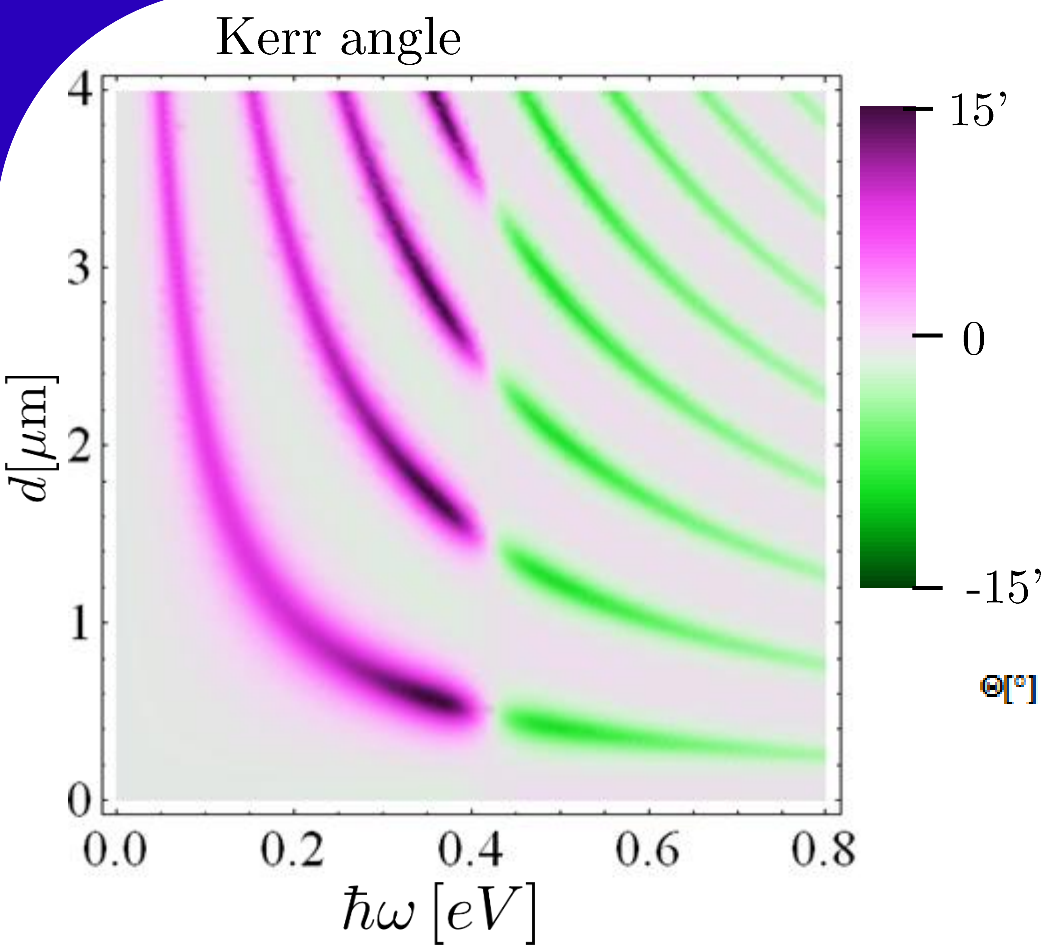
Our calculation

E. V. Gorbar, et al., PRB (2012)

R. Nandkishore, et al., PRL (2011)

Differences?

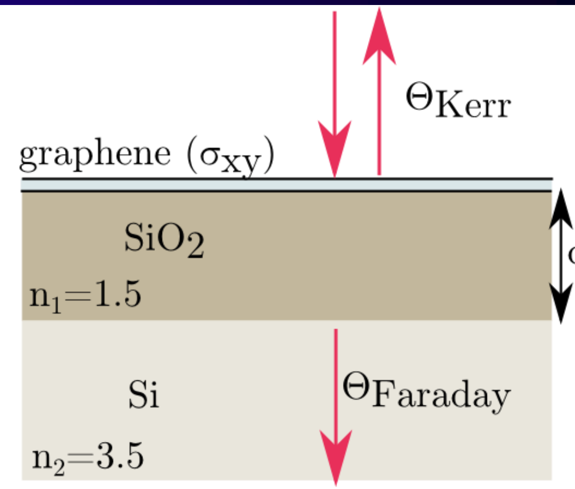
Kerr angle in bilayer graphene

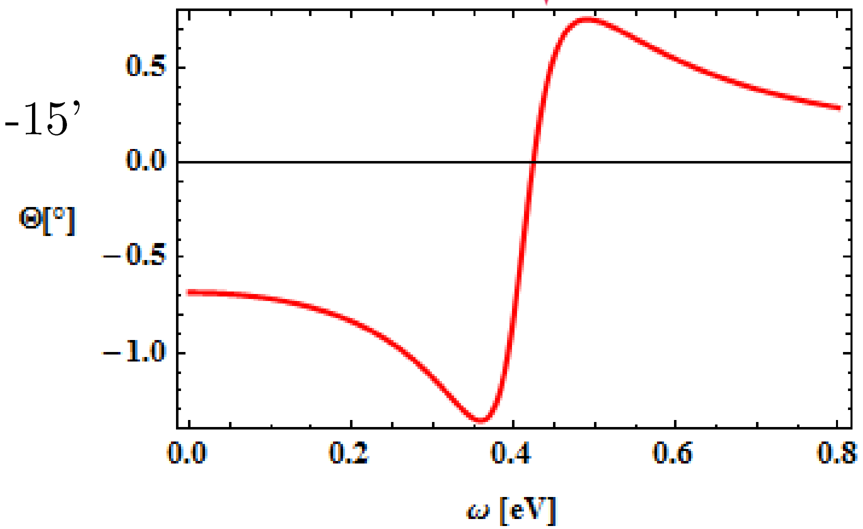


Large Kerr angle lines:

$$d = \frac{\pi}{2k} + \frac{n\pi}{k} \qquad n \in \mathbb{N}$$

$$\Theta_{Kerr} = \frac{2n_2^2}{n_2^2 - n_1^4} \mathrm{Re}\left[\frac{\sigma_{xy}}{\epsilon_0 c}\right]$$





free-standing graphene

Experimentally relevant regime, sensitivity 0.001 deg

Conclusion

- Practical method to calculate Kerr angle in layered nanostructures
  - Transfer matrix method
- Optical Hall conductivity
  - Bilayer graphene
  - Broken symmetry
- Example: SiO_2, Si substrate
- Spectroscopic method of identifying phase of bilayer graphene

Acknowledgement:

József Cserti

Thanks for your attention!