Tailoring topological superconductivity using supercurrents

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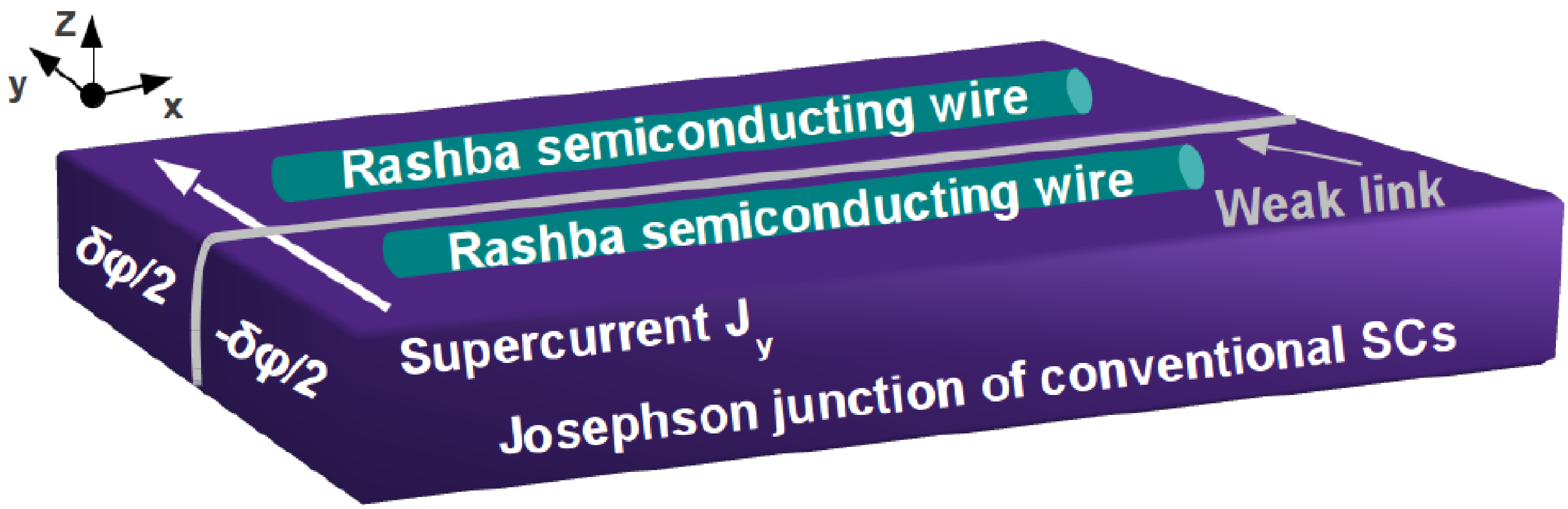
Overview

- Quick look at experiments

- Classification of engineered TSCs

- Our proposal based on supercurrents:

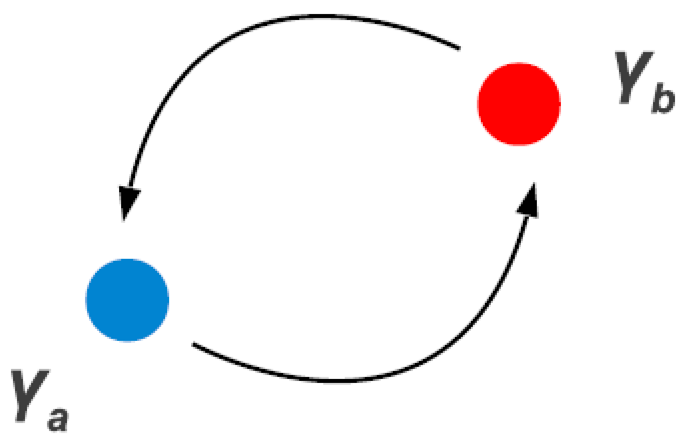
  MFs in quasi-1d Rashba semiconductors +SC + supercurrents



Majorana fermions and TQC

Topological qubits (**TQ**) are in principle immune to decoherence and noise compared to conventional spin and superconducting qubits

Typical braiding process



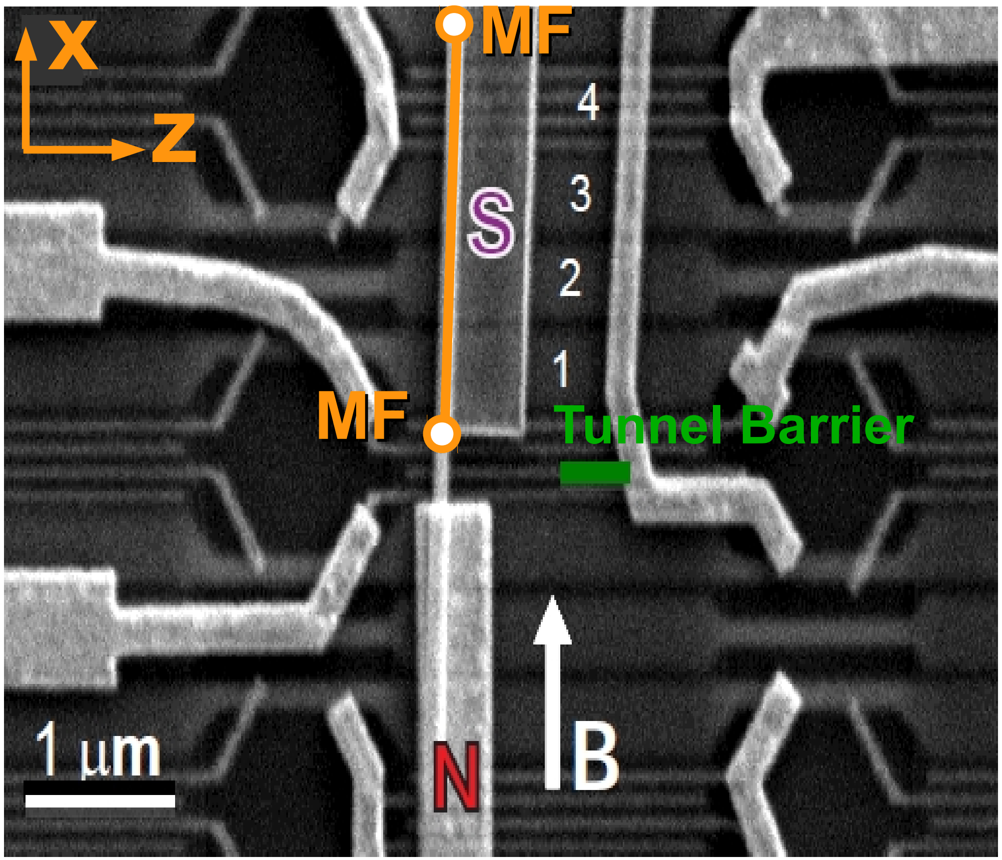
$$d = \frac{\gamma_a + i\gamma_b}{\sqrt{2}}$$

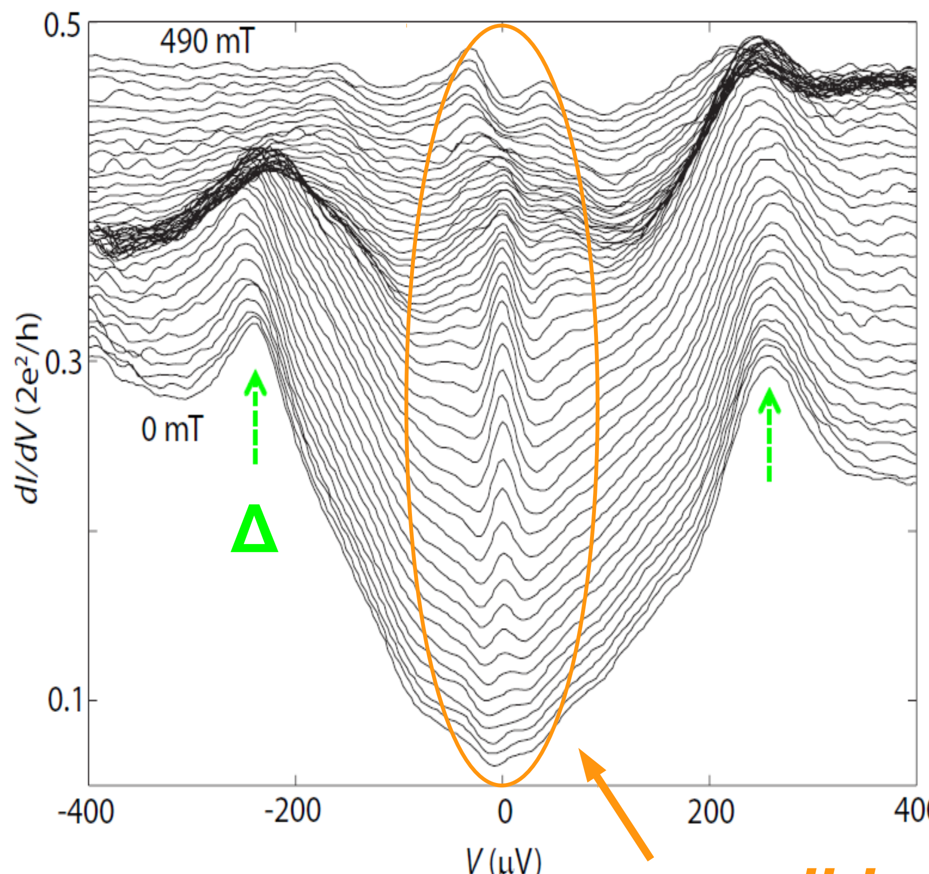
$$\gamma_a \to +\gamma_b$$
$$\gamma_b \to -\gamma_a$$

$$\left(\left|1\right\rangle, \left|0\right\rangle\right) \to \left(e^{i\pi/4}\left|1\right\rangle, e^{-i\pi/4}\left|0\right\rangle\right)$$

The Delft experiment

Topological superconductor $\mu_s \mathcal{B} > \sqrt{\Delta^2 + \mu^2}$ ***Lutchyn et al., Oreg et al. PRL 2010***





Mourik et al., Science 2012

Other experiments

Rokhinson, Xinyu and Furdyna Nat. Phys. 2012

Deng et al. Nanoletters 2012

Das et al. Nat. Phys. 2012

Williams et al. PRL 2012

Churchill et al. PRB 2013

MFs seem to be too easy to get....

How can we access MF physics?

Kinetic term+chemical potential **S-O coupling** **Zeeman field** **SC order parameter**

$$\mathcal{H}_k = \left[\frac{(\hbar k)^2}{2m} - \mu\right]\tau_z + v\hbar k\sigma_z - \mu_s\mathcal{B}\tau_z\sigma_x - \Delta\tau_y\sigma_y$$

$$\Psi_k^\dagger = \begin{pmatrix} c_{k\uparrow}^\dagger & c_{k\downarrow}^\dagger & c_{-k\uparrow} & c_{-k\downarrow} \end{pmatrix}$$

TSC wire Hamiltonian

BDI symmetry class

Bogoliubov eigenoperators are **only** of the Majorana type:

$$\gamma_k^\dagger = \gamma_{-k}$$

Note that MFs

$$\gamma_k^\dagger = \gamma_k$$

How can we access MF physics?

What is the crucial effect of the perpendicular magnetic field?

If $\mathcal{B} = 0$ we can use the 2-component Nambu spinor

$$\Psi_k^\dagger = \begin{pmatrix} c_{k\uparrow}^\dagger & c_{-k\downarrow} \end{pmatrix}$$

$$\mathcal{H}_k = \left[\frac{(\hbar k)^2}{2m} - \mu + v\hbar k\right]\tau_z + \Delta\tau_x$$

AIII symmetry class

Z top. invariant in 1d

Bogoliubov eigenoperators are **not** of the Majorana type

The Majorana picture is for this type of TSCs,
an equivalent but unnecessary description

Periodic table of topological systems

AZ	Time reversal	Charge conjugation	Chiral symmetry	d = 1	2	3	4	5	6	7	8
BDI	1	1	1	$\mathbb{Z}$	0	0					
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0					
DIII	−1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$					

Classes which support only MFs

$$[\widehat{\mathcal{H}}(\hat{p},r),\Theta]=0 \Rightarrow \Theta^{-1}\widehat{\mathcal{H}}(\hat{p},r)\Theta=+\widehat{\mathcal{H}}(\hat{p},r)$$

$$\{\widehat{\mathcal{H}}(\hat{p},r),\Xi\}=0 \Rightarrow \Xi^{-1}\widehat{\mathcal{H}}(\hat{p},r)\Xi=-\widehat{\mathcal{H}}(\hat{p},r)$$

$$\{\widehat{\mathcal{H}}(\hat{p},r),\Theta\Xi\}=\{\widehat{\mathcal{H}}(\hat{p},r),\Pi\}=0$$

Microscopic model for engineered TSCs

$$\mathcal{H} = \int dr\ \hat{\psi}^\dagger(r) \left[\frac{\hat{p}^2}{2m} - \mu + V(r) - M(r) \cdot \sigma \right] \hat{\psi}(r)$$

$$+ \int dr\ \ \hat{\psi}^\dagger(r) \frac{\{v(r), \hat{p}_x \sigma_y - \hat{p}_y \sigma_x\}}{2} \hat{\psi}(r)$$

$$+ \int dr \left[\psi_\uparrow^\dagger(r) \Delta(r) \psi_\downarrow^\dagger(r) + \psi_\downarrow(r) \Delta^*(r) \psi_\uparrow(r) \right]$$

$$\hat{\psi}^\dagger(r) = (\psi_\uparrow^\dagger(r)\,, \psi_\downarrow^\dagger(r))$$

Landscape of Topological SC Phases

PK arXiv:1305.0131, to appear in NJP

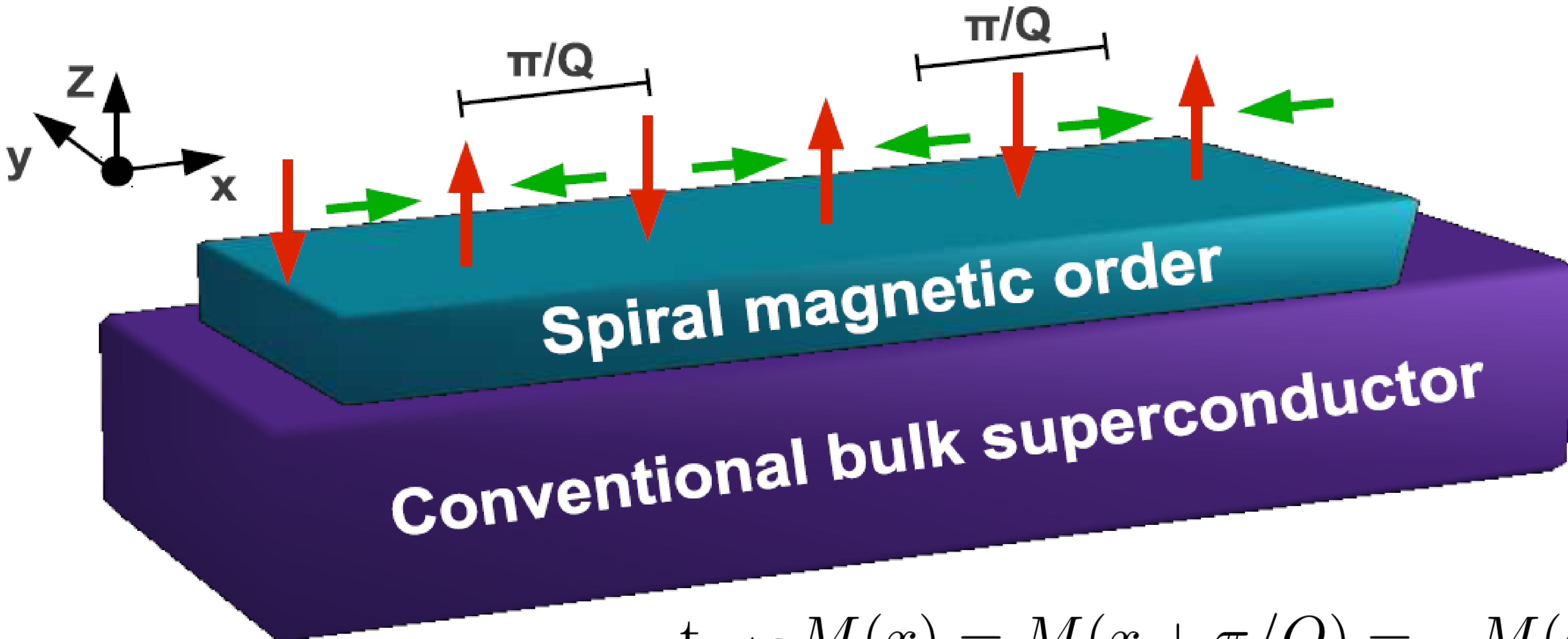
Case	$v(\boldsymbol{r})$	$M(\boldsymbol{r})$	$\Delta(\boldsymbol{r})$	2d	quasi-1d	1d
I	✓	×	$\mathcal{K} = \mathrm{I}$	**DIII**	**DIII**	*no MFs*
II	✓	×	$\mathcal{K} = 0$	**D**	**D**	*no MFs*
III	×	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	*BDI*	**BDI**	**BDI**
IV	×	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**
V	✓	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	**D**	**D**	**BDI**
VI	✓	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**

Standard model TSC wires (Lutchyn, Oreg, Romito)

Random magnetism + superconductivity (Choy et al.)
Helical magnetism + superconductivity (Martin, Flensberg)
More recently (Yazdani, Nagaosa, Loss, Simon, Franz, Glazman)

SC-TI-SC linear junction (Fu and Kane)

Helical magnetism + SC



$$\mathrm{t}_{\pi/Q} M(x) = M(x+\pi/Q) = -M(x)$$

The Hamiltonian is invariant under the operation $\mathrm{O}_a = t_{\pi/Q}\mathcal{T}$

When $M_y = 0$ the Hamiltonian is invariant under $\mathcal{K}$ and $\mathcal{O}_a\mathcal{K}$

Symmetry class: $\mathrm{BDI} \oplus \mathrm{BDI}$ ***PK 2013***

When $M_y \neq 0$ the Hamiltonian belongs to class BDI

Topological SC Phases: New directions

Case	$v(\boldsymbol{r})$	$M(\boldsymbol{r})$	$\Delta(\boldsymbol{r})$	2d	quasi-1d	1d
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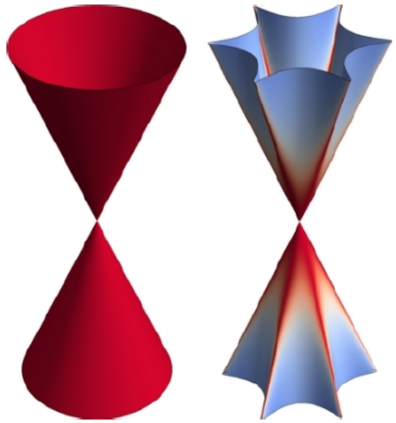
III	×	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	*BDI*	**BDI**	**BDI**
IV	×	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**
V	✓	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	**D**	**D**	**BDI**
VI	✓	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**

Hexagonally warped TI surface states + magnetism + SC



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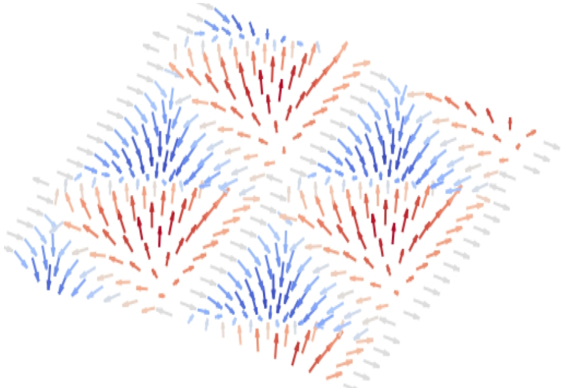

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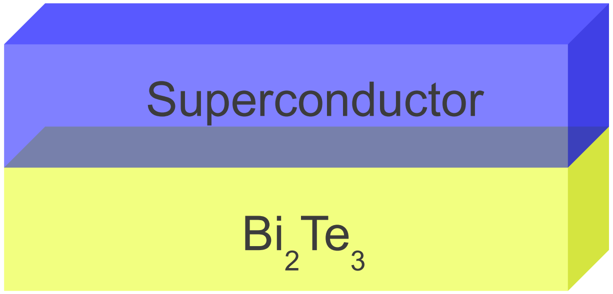


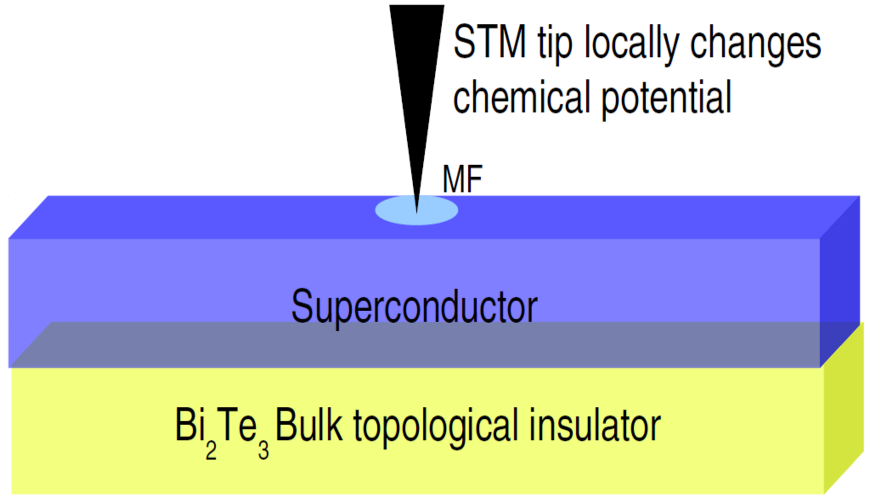
Daniel Mendler's poster

Topological SC Phases: New directions

Case	$v(\boldsymbol{r})$	$\boldsymbol{M}(\boldsymbol{r})$	$\Delta(\boldsymbol{r})$	2d	quasi-1d	1d
I	✓	×	$\mathcal{K} = \mathrm{I}$	**DIII**	**DIII**	*no MFs*
II	✓	×	$\mathcal{K} = 0$	**D**	**D**	*no MFs*
III	×	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	*BDI*	**BDI**	**BDI**
IV	×	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**
V	✓	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	**D**	**D**	**BDI**
VI	✓	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**

A canvas for TQC using STM-tips

Easier to handle compared to networks of wires

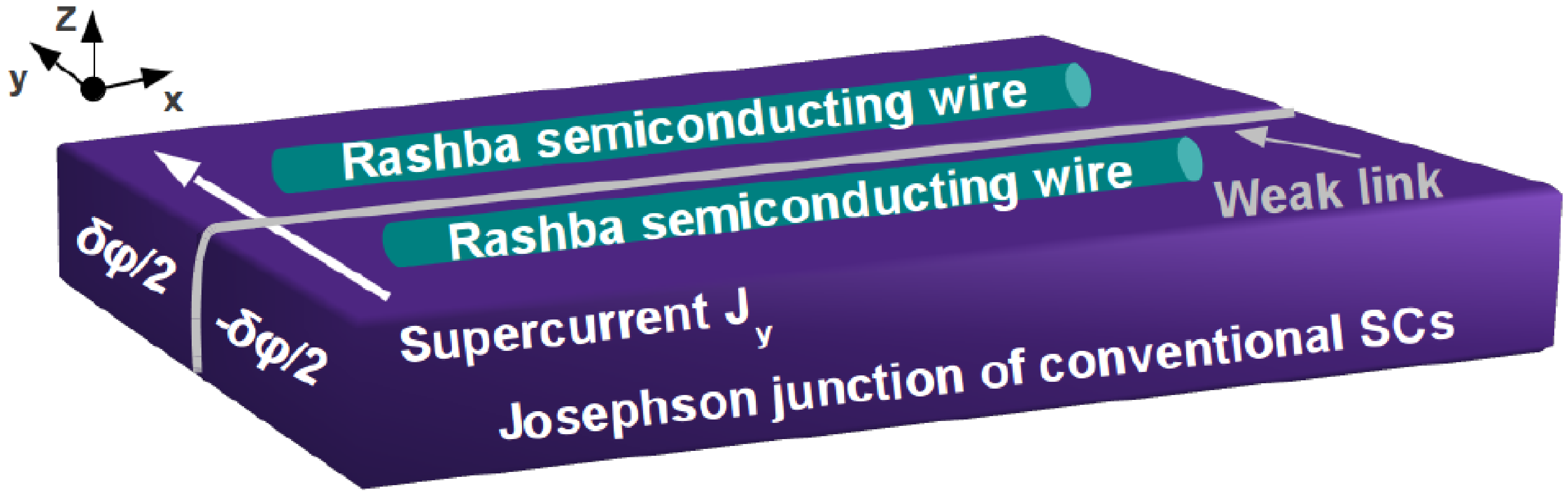


Heterostructure with STM tip

Topological SC Phases: New directions

Case	$v(\boldsymbol{r})$	$\boldsymbol{M}(\boldsymbol{r})$	$\Delta(\boldsymbol{r})$	2d	quasi-1d	1d
I	✓	×	$\mathcal{K} = \mathrm{I}$	**DIII**	**DIII**	*no MFs*
II	✓	×	$\mathcal{K} = 0$	**D**	**D**	*no MFs*
III	×	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	*BDI*	**BDI**	**BDI**
IV	×	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**
V	✓	$\mathcal{K} = \mathrm{I}$	$\mathcal{K} = \mathrm{I}$	**D**	**D**	**BDI**
VI	✓	$\mathcal{K} = \{0, \mathrm{I}, 0\}$	$\mathcal{K} = \{\mathrm{I}, 0, 0\}$	**D**	**D**	**D**

Quasi-1d Rashba semiconductor + SC + supercurrent



MFs in a double Rashba wire setup + SC + J

Inter-wire hopping | Intra-wire spin-orbit | Inter-wire spin-orbit

$$\widehat{\mathcal{H}}(\hat{p}_x) = \left(\frac{\hat{p}_x^2}{2m} - \mu\right)\tau_z + \underline{t_\perp \tau_z \kappa_x} + \underline{v\hat{p}_x \tau_z \sigma_y} - \underline{V_\perp \kappa_y \sigma_x}$$

$$\underline{-e^{i\frac{\delta\varphi}{2}\tau_z\kappa_z}}\,\underline{\Delta\tau_y\sigma_y} - \Delta_\perp \underline{\tau_y\kappa_x}\sigma_y$$

supercurrent | Intra-wire SC | Inter-wire SC

$$\widehat{\Psi}^\dagger(x) = (\psi^\dagger_{\uparrow,+}(x)\,,\psi^\dagger_{\downarrow,+}(x)\,,\psi^\dagger_{\uparrow,-}(x)\,,\psi^\dagger_{\downarrow,-}(x),$$
$$\psi_{\uparrow,+}(x)\,,\psi_{\downarrow,+}(x)\,,\psi_{\uparrow,-}(x)\,,\psi_{\downarrow,-}(x))$$

Gauging away the supercurrent

$$\widehat{\mathcal{H}}(\hat{p}_x) = \left(\frac{\hat{p}_x^2}{2m} - \mu\right)\tau_z + v\hat{p}_x\tau_z\sigma_y - V_\perp e^{-i\frac{\delta\varphi}{2}\tau_z\kappa_z}\kappa_y\sigma_x - \Delta\tau_y\sigma_y$$

$$-\Delta_\perp\tau_y\kappa_x\sigma_y + t_\perp e^{-i\frac{\delta\varphi}{2}\tau_z\kappa_z}\tau_z\kappa_x$$

For a π-junction:

$$\widehat{\mathcal{H}}(\hat{p}_x) = \left(\frac{\hat{p}_x^2}{2m} - \mu\right)\tau_z + v\hat{p}_x\tau_z\sigma_y + V_\perp\tau_z\kappa_x\sigma_x - \Delta\tau_y\sigma_y$$

$$-\Delta_\perp\tau_y\kappa_x\sigma_y + t_\perp\kappa_y$$

Inter-wire SO →
Inter-wire Zeeman field

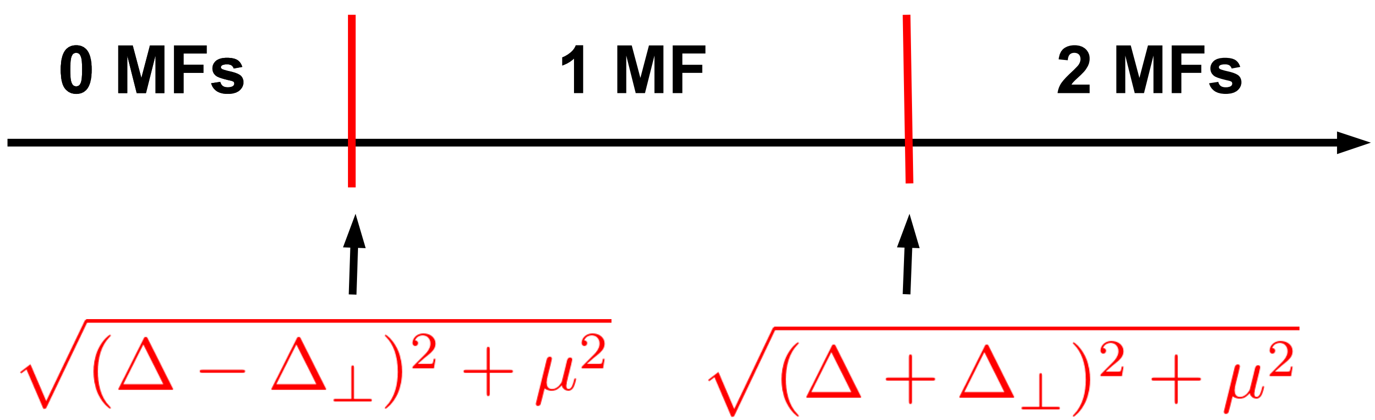
The special case of mirror symmetry

For a π-junction and zero inter-wire hopping $t_\perp = 0$ we obtain

$$\widehat{\mathcal{H}}_\kappa(\hat{p}_x) = \left(\frac{\hat{p}_x^2}{2m} - \mu\right)\tau_z + v\hat{p}_x\tau_z\sigma_y + \kappa V_\perp \tau_z \kappa_x \sigma_x$$

$$-(\Delta + \kappa\Delta_\perp)\tau_y\sigma_y \qquad \mathrm{BDI} \oplus \mathrm{BDI}$$

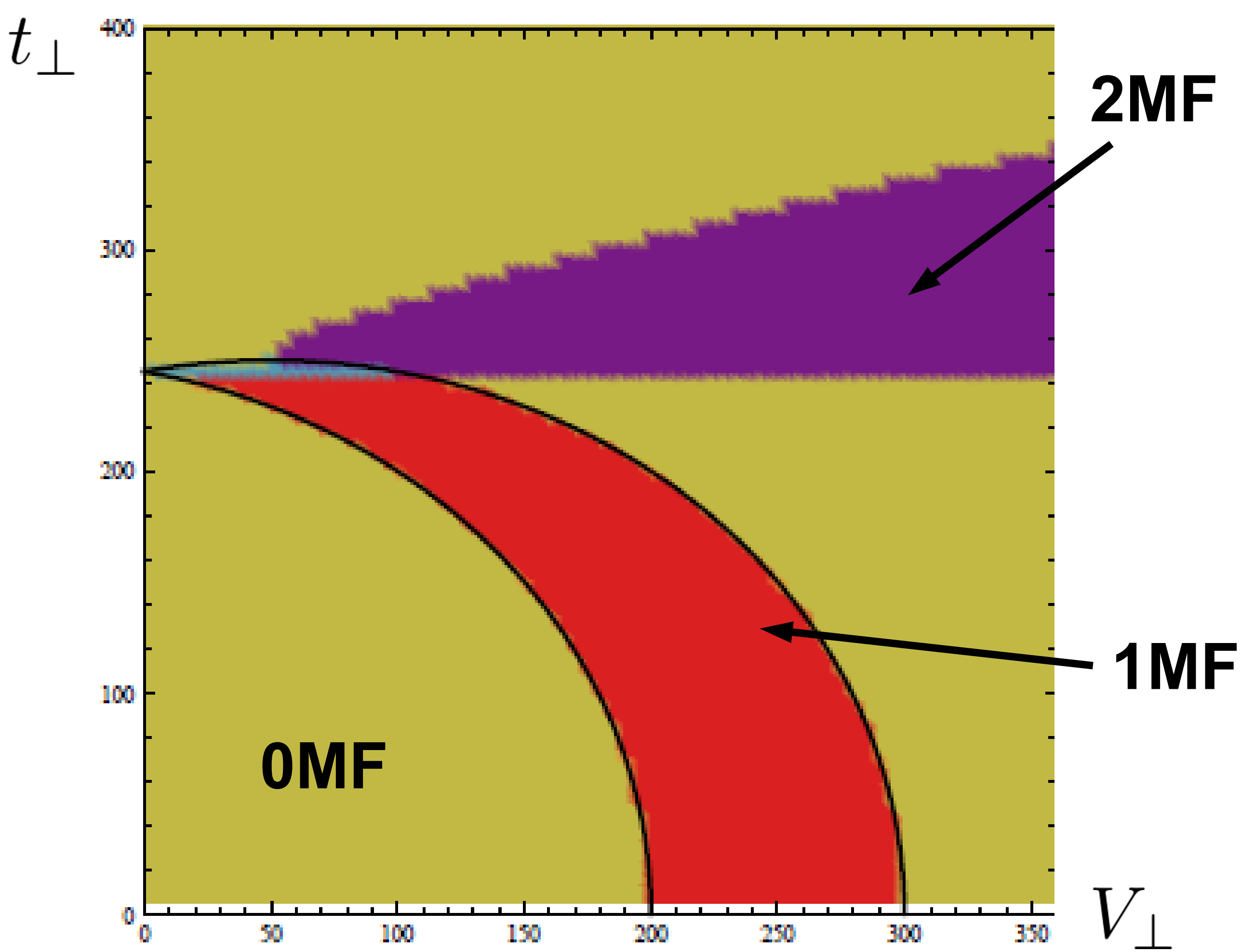
Two-decoupled TSC wire Hamiltonians similar to those in the Delft experiment because of mirror symmetry

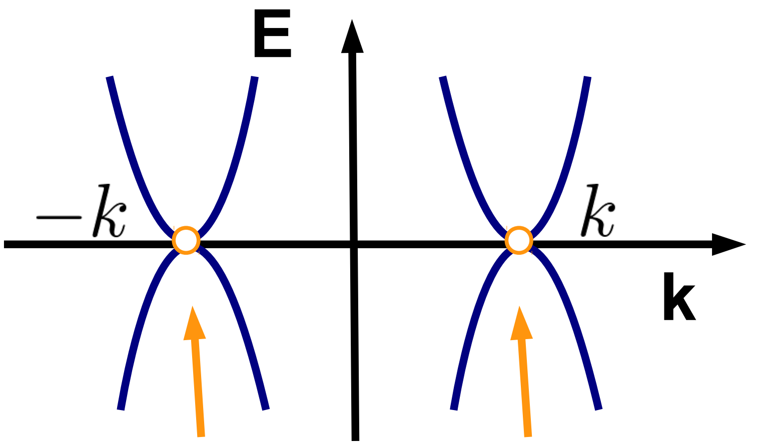


General case – Topological phase diagram

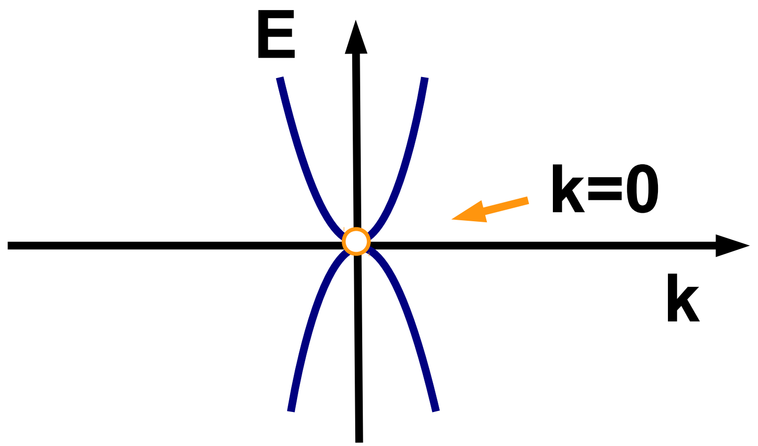
Symmetry class: BDI **Chiral symmetry:** $\tau_x \kappa_x$

Z topological invariant (Schnyder et al., Sau and Tewari)





Gapped if BDI → D,
e.g. intra-wire SC asymmetry



Conclusions and new perspectives

- A complete classification of 2d, quasi-1d and 1d systems with inhmomogeneous magnetism, Rashba s-o coupling and SC.
- Emergent symmetries are crucial for the topological properties of the system.
- Quasi-1d systems offer new possibilities **without** the requirement of any kind of **magnetism**.
- Two-channel / Two-wire Rashba semiconductors + SC + J
- Other possibilities involving simultaneously supercurrents and magnetism

Thank you...