Hardwiring Maxwell's Demon

Tobias Brandes (Institut für Theoretische Physik, TU Berlin)

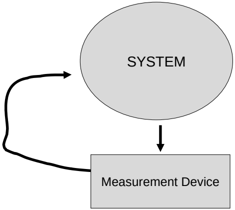
- Introduction: feedback loops.
- Feedback loops in transport
  - ' by hand'.
  - ' by hardwiring': thermoelectric device.
- Maxwell demon limit.

Co-workers:
Gernot Schaller, Philipp Strasberg (TU Berlin)

Massimiliano Esposito (Luxembourg).

Introduction

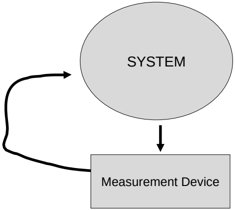
Closed loop (feedback) control



- System parameters are permanently changed, conditioned on measurement result.

Introduction

Closed loop (feedback) control



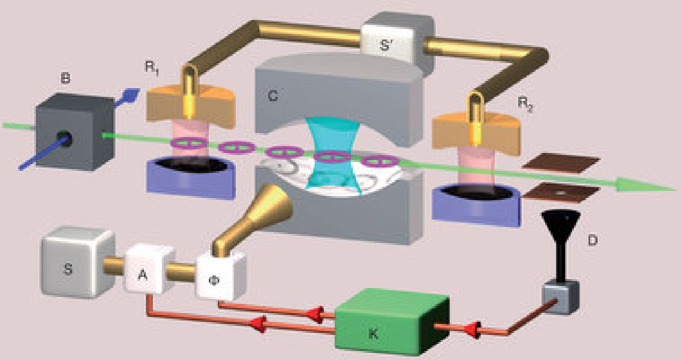
- System parameters are permanently changed, conditioned on measurement result.

- Experiments so far: classical systems. Quantum optics.
- Goal: feedback control of quantum transport.

Introduction

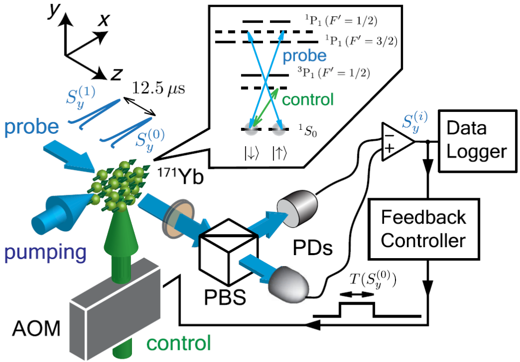
Real-time quantum feedback

- Prepares and stabilizes photon number states



C. Sayrin, I. Dotsenko, X. Zhou, B. Peaudecerf, T. Rybarczyk, S. Gleyzes, P. Rouchon, M. Mirrahimi, H. Amini, M. Brune, J.-M. Raimond, S. Haroche; Nature **477**, 73 (2011).

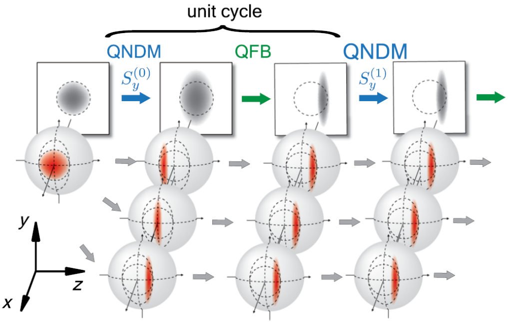
Introduction



R. Inoue, S. Takana, R. Namiki, T. Sagawa, Y. Takahashi, PRL **110**, 163602 (2013).

- N two-level systems. Collective spin $\hat{J}_z \equiv \frac{1}{2}\sum_{i=1}^{N} \hat{\sigma}_z^{(i)}$, $[\hat{J}_y, \hat{J}_z] = i\hat{J}_x$.
- Heisenberg uncertainty relation $\delta J_y \delta J_z \geq |J_x|/2$

Introduction



- Measurement (Faraday rotation) $\rightsquigarrow$ squeezing.

State-dependent feedback control:

- Control pulse compensates for random shift.

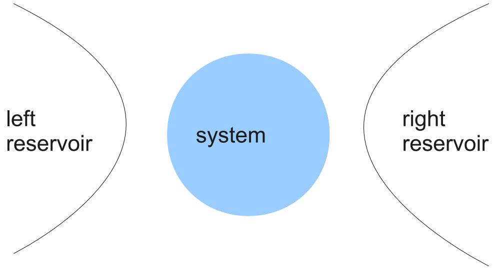
R. Inoue, S. Takana, R. Namiki, T. Sagawa, Y. Takahashi, PRL **110**, 163602 (2013).

Feedback control of transport through nanostructures

Master Equation

General setup

- Open system Hamiltonian. $\mathcal{H} = \mathcal{H}_S + \mathcal{H}_{\rm res} + \mathcal{H}_T$.
  - $\mathcal{H}_S$ system.
  - $\mathcal{H}_{\rm res}$ reservoir.
  - $\mathcal{H}_T$ system-reservoir coupling.
- Reduced density matrix $\rho(t)$, Liouvillian $\mathcal{L}$, Born-Markov approximation $\dot{\rho}(t) = \mathcal{L}\rho(t), \quad \mathcal{L} \equiv \mathcal{L}_0 + \mathcal{J}$



Master Equation

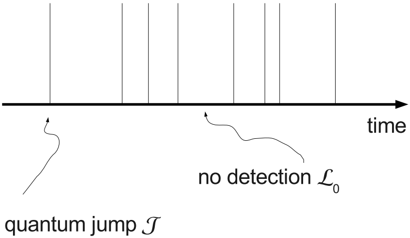
Quantum jumps

Jump-resolved ('n-resolved') Master equation

$$\rho(t) = \sum_{n=0}^{\infty} \rho^n(t) = \sum_{n=0}^{\infty} \int_0^t dt_n ... \int_0^{t_2} dt_1 \rho^c(t; t_n, ..., t_1)$$

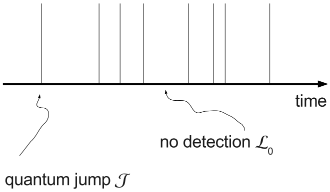
$$\rho^c(t; t_n, ..., t_1) \equiv e^{\mathcal{L}_0 \cdot (t - t_n)} \mathcal{J} e^{\mathcal{L}_0 \cdot (t_n - t_{n-1})} \mathcal{J} ... \mathcal{J} e^{\mathcal{L}_0 \cdot t_1} \rho_0$$

- Non-unitary free time-evolution, interrupted by n quantum jumps at times t_i.



Master Equation

Feedback conditioned on quantum jumps

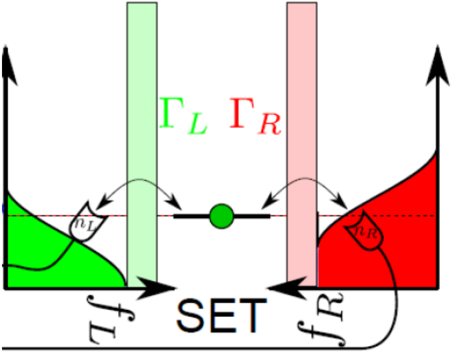


- Scheme 1
  - Rotate state vector by upgrading counting field χ in $\dot{\rho} = (\mathcal{L}_0 + e^{i\chi}\mathcal{J})\rho$ to superoperator $\mathcal{K}$, H. Wiseman, G. Milburn (1990s).
- **Scheme 2**
  - Effectively modulate system reservoir coupling $\rightsquigarrow$ Maxwell demon.

Feedback controlled tunnel barrier

Single electron transistor

- Modify tunnel rates, e.g. Γ_R, depending on dot occupation.

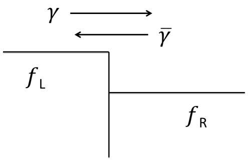


G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).

Feedback controlled tunnel barrier

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.

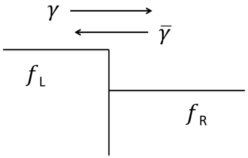


$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

Feedback controlled tunnel barrier

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L(1-f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1-f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.



$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

Without feedback:

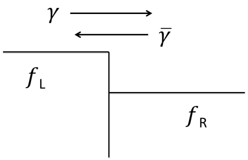
- Identical microscopic forward and backward rates $\Gamma = \bar{\Gamma}$.
- Local **detailed balance** condition with affinity $\mathcal{A}$,

$$\frac{\gamma}{\bar{\gamma}} = e^{\mathcal{A}}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}$$

Feedback controlled tunnel barrier

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.



$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

With feedback:

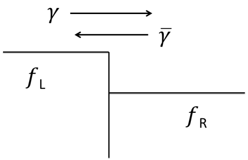
- Different forward and backward rates $\Gamma \neq \bar{\Gamma}$ ('by hand')
- Violates local detailed balance condition,

$$\frac{\gamma}{\bar{\gamma}} = e^{\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}$$

Feedback controlled tunnel barrier

Single junction

- Integrate out the dot for $\Gamma_L \gg \Gamma_R \equiv \Gamma$.
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.
- $p_n(t)$ probability for n charges transfered to right reservoir.



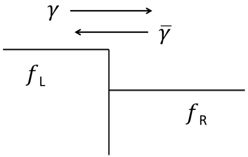
$$\dot{p}_n = \gamma p_{n-1} + \bar{\gamma} p_{n+1} - (\gamma + \bar{\gamma}) p_n$$

- Elevate (violated) local detailed balance to modified *exchange fluctuation theorem*

$$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln\frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Feedback controlled tunnel barrier

Single junction: Interpretation



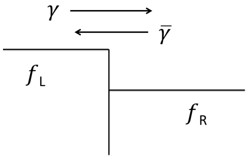
- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.

$$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right) n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

- Stationary charge current $\mathcal{J} = \gamma - \bar{\gamma}$ (set $-e = 1$).
- With feedback $\Gamma \neq \bar{\Gamma} \rightsquigarrow$ finite $\mathcal{J}$ even for zero voltage drop.

Feedback controlled tunnel barrier

Single junction: Interpretation



- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.

$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}$.

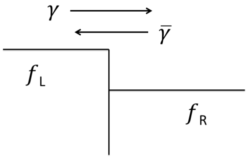
- Only n (number of transfered charges) thermodyn. relevant.
- Shannon entropy $S \equiv -\sum_n p_n \ln p_n$.
- Decompose $\dot{S} = \dot{S}_e + \dot{S}_i$ with $\dot{S}_i \geq 0$, J. Schnakenberg, Rev. Mod. Phys. **48**, 571 (1976); M. Esposito, C. Van den Broek, Phys. Rev. E **82**, 011143 (2010).

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\dot{S}_i t}, \quad \dot{S}_i = \mathcal{A}\mathcal{J} + \ln \frac{\Gamma}{\bar{\Gamma}} \mathcal{J}, \quad \mathcal{J} \equiv \gamma - \bar{\gamma}$$

$\dot{S}_i$ = dissipated electric power per $k_B T$ plus *information current*.

Feedback controlled tunnel barrier

Single junction: Interpretation



- Rates $\gamma = \Gamma f_L(1 - f_R)$, $\bar{\gamma} = \bar{\Gamma} f_R(1 - f_L)$.

$\frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln \frac{\Gamma}{\bar{\Gamma}}\right)n}, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$

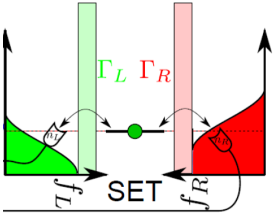
- Information gain via feedback modifies exchange fluctuation relation.
- General scenario T. Sagawa and M. Ueda, Phys. Rev. Lett. **104**, 090602 (2010);...; H. Tasaki arXiv:1308.3776
- Example $\langle e^{-\beta(W - \Delta F) - I} \rangle = 1$ Jarzynski.

Feedback controlled tunnel barrier

Single electron transistor

- Rate equation $\dot{\rho} = \mathcal{L}\rho,\ \rho = (\rho_0, \rho_1)^T$.
- Explicitely break local detailed balance:

$$\mathcal{L} = \sum_{\alpha=L,R} \begin{pmatrix} -\Gamma_\alpha f_\alpha & \bar{\Gamma}_\alpha(1-f_\alpha)e^{i\chi} \\ \Gamma_\alpha f_\alpha & -\bar{\Gamma}_\alpha(1-f_\alpha) \end{pmatrix}$$



G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).

- Fluctuation relation (affinity $\mathcal{A} \equiv V/k_B T$, voltage $V \equiv \mu_L - \mu_R$)

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A} + \ln\frac{\Gamma_R}{\bar{\Gamma}_R} + \ln\frac{\bar{\Gamma}_L}{\Gamma_L}\right)n}.$$

M. Esposito, G. Schaller; EPL **99**, 30003 (2012).

Feedback controlled tunnel barrier

Single electron transistor

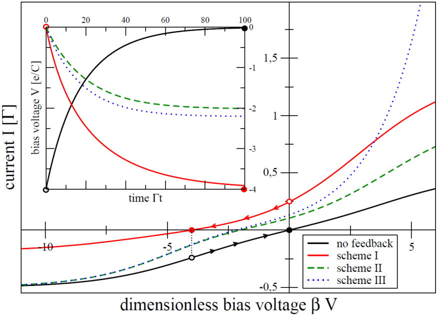
- Fluctuation relation (affinity $\mathcal{A} \equiv V/k_B T$, voltage $V \equiv \mu_L - \mu_R$)

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = e^{\left(\mathcal{A}+\ln\frac{\Gamma_R}{\bar{\Gamma}_R}+\ln\frac{\bar{\Gamma}_L}{\Gamma_L}\right)n}.$$

M. Esposito, G. Schaller; EPL **99**, 30003 (2012).

- Term $\ln\frac{\Gamma_R}{\bar{\Gamma}_R}+\ln\frac{\bar{\Gamma}_L}{\Gamma_L} = -V^*/k_B T$ as offset-voltage

G. Schaller, C. Emary, G. Kießlich, TB; Phys. Rev. B **84**, 085418 (2011).



Feedback controlled tunnel barrier

Summary up to here

- Concept of a transport device that acts like Maxwell's demon.

Maxwell's demon

A feedback mechanism that shuffles particles against a chemical or thermal gradient by adjusting barriers without doing work.

Colloquium: The physics of Maxwell's demon and information; K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. **81**, 1 (2009).

- System energies are not changed.

Feedback controlled tunnel barrier

Summary up to here

- Concept of a transport device that acts like Maxwell's demon.

Maxwell's demon

A feedback mechanism that shuffles particles against a chemical or thermal gradient by adjusting barriers without doing work.

Colloquium: The physics of Maxwell's demon and information; K. Maruyama, F. Nori, and V. Vedral, Rev. Mod. Phys. **81**, 1 (2009).

- System energies are not changed.
- Feedback loop in our SET model: $\Gamma_\alpha \neq \bar{\Gamma}_\alpha$ 'by hand'.

Hardwiring the demon

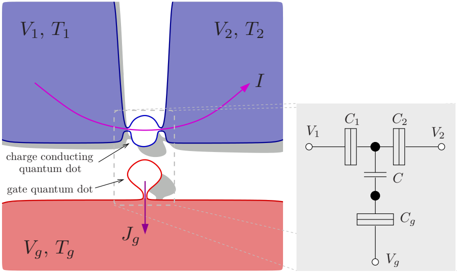
Key idea

- Microscopic model for larger system : SET + detector.
- Reduced SET dynamics described by effective model as above.

Hardwiring the demon

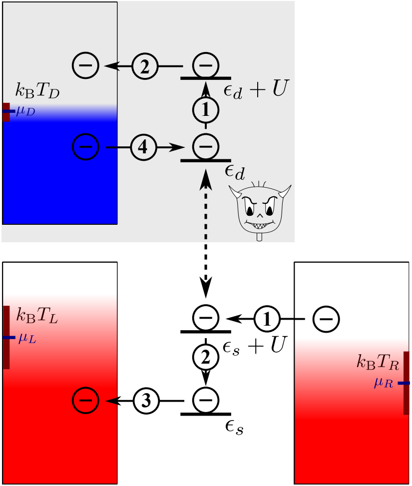
Thermoelectric device

- Energy to current converter.
- Different temperatures in different parts of the system.
- Energy-dependent tunnel rates.



R. Sánchez, M. Büttiker, Phys. Rev. B **83**, 085428 (2011); Europhys. Lett. **100**, 47008 (2012).

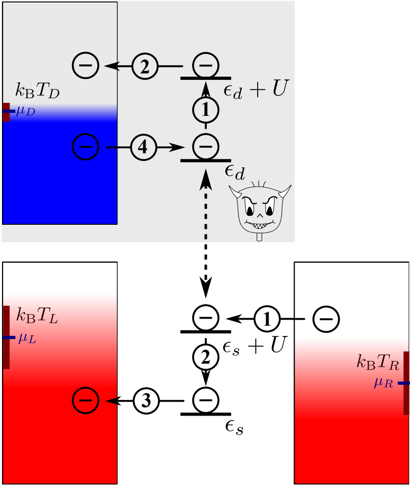
Hardwiring the demon

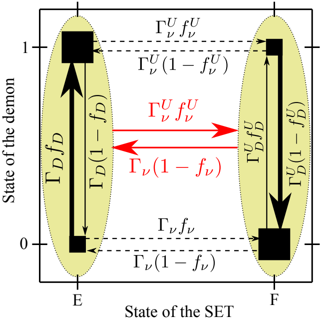


P. Strasberg, G. Schaller, TB, M. Esposito,
Phys. Rev. Lett. **110**, 040601 (2013).

- Single level SET (bottom, two reservoirs L/R) and detector (top, one reservoir).
- States $|0E\rangle$, $|0F\rangle$, $|1E\rangle$, $|1F\rangle$.
- Energies 0, ϵ_s, ϵ_d, $\epsilon_s + \epsilon_d + U$.
- Energy dependent rates.

Hardwiring the demon





P. Strasberg, G. Schaller, TB, M. Esposito, Phys. Rev. Lett. **110**, 040601 (2013).

- Detector requirements:
  - Fast $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Precise $U \gg k_B T_D$.
- SET requirements:
  - Spatial asymmetry $\Gamma_R^U \gg \Gamma_L^U$, $\Gamma_L \gg \Gamma_R$.

Maxwell demon limit

Reduced SET dynamics

- Full rate equation $\dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'}$
- Separation of time scales for fast detector $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E **85**, 041125 (2012).
- $\rightsquigarrow$ SET rate equation $\dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'}$.

Maxwell demon limit

Reduced SET dynamics

- Full rate equation $\dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'}$
- Separation of time scales for fast detector $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E **85**, 041125 (2012).
- $\rightsquigarrow$ SET rate equation $\dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'}$.

Reduced fluctuation theorem for SET with information current I

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = \exp\left[\mathcal{A}n + I \times t\right], \quad I \times t \equiv \ln \frac{f_L^U f_R \Gamma_L^U \Gamma_R}{f_R^U f_L \Gamma_R^U \Gamma_L} n, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Maxwell demon limit

Reduced SET dynamics

- Full rate equation $\dot{\rho}_{ij} = \sum_{i'j'} W_{ij,i'j'} \rho_{i'j'}$
- Separation of time scales for fast detector $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
  - Technical trick: use conditional stationary probabilities M. Esposito, Phys. Rev. E **85**, 041125 (2012).
- $\rightsquigarrow$ SET rate equation $\dot{\rho}_i = \sum_{i'} V_{ii'} \rho_{i'}$.

Reduced fluctuation theorem for SET with information current I

$$\lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} = \exp\left[\mathcal{A}n + I \times t\right], \quad I \times t \equiv \ln \frac{f_L^U f_R \Gamma_L^U \Gamma_R}{f_R^U f_L \Gamma_R^U \Gamma_L} n, \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

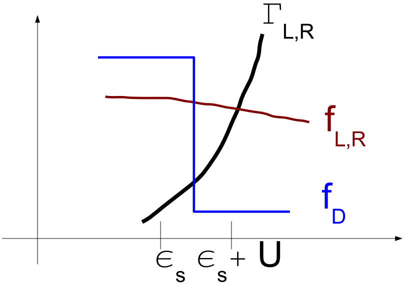
- Reduction to previous model ('feedback by hand'):

$$f_\alpha^U / f_\alpha = 1 \rightsquigarrow I \times t = (\ln \Gamma_L^U \Gamma_R / \Gamma_R^U \Gamma_L) n$$

Maxwell demon limit

Summary of demon conditions

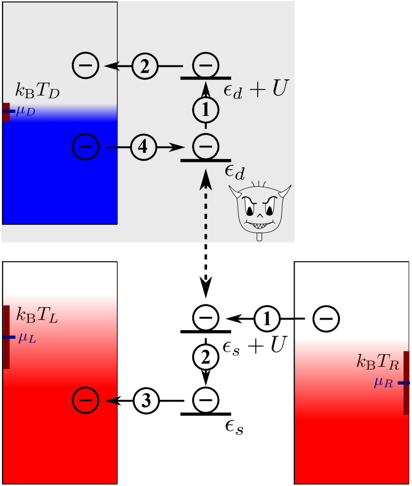
- Separation of time scales:
  - Fast demon $\Gamma_D, \Gamma_D^U \gg \Gamma_\alpha, \Gamma_\alpha^U$.
- Separation of energy scales:
  - Precise demon $U \gg k_B T_D$.
  - Almost no back-action $k_B T \gg U$.
  - Good energy lever $\Gamma_\alpha \neq \Gamma_\alpha^U$, $\alpha =$ L,R.



$$\rightsquigarrow \lim_{t\to\infty} \frac{p_n(t)}{p_{-n}(t)} \approx \exp\left[\left(\mathcal{A} + \ln \frac{\Gamma_L^U \Gamma_R}{\Gamma_R^U \Gamma_L}\right) n\right], \quad \mathcal{A} \equiv \frac{\mu_L - \mu_R}{k_B T}.$$

Maxwell demon limit

Energetics

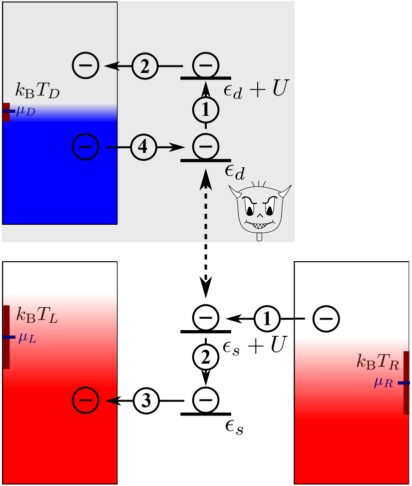


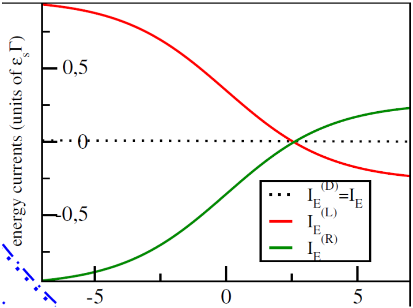
- Cycle between with system energies: $\epsilon_d \to \epsilon_s + \epsilon_d + U \to \epsilon_s \to 0 \to \epsilon_d$.
- Net energy U transferred from SET to detector.
- 'First law' for energy currents $I_L^E + I_R^E + I_D^E = 0$.

Demon limit: $I_L^E + I_R^E \approx 0$ for $U \ll k_B T$.

Maxwell demon limit

Energetics



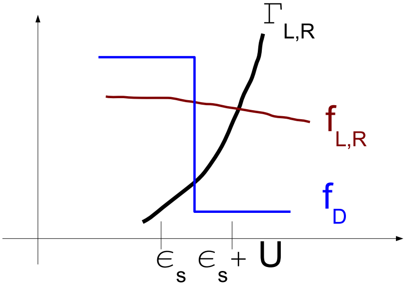


Maxwell demon limit

Where is the demon?

Maxwell's demon

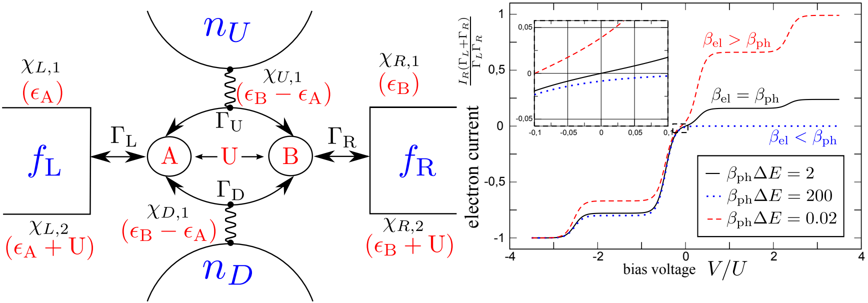
A feedback mechanism that shuffles particles against a chemical or thermal gradient by adjusting barriers without requiring work.



- 'Hardwiring' of the feedback mechanism.
- 'Information' is really physical: rates in term $\ln\left(\Gamma_L^U \Gamma_R / \Gamma_R^U \Gamma_L\right)$.

Alternative scheme

Model with phonons



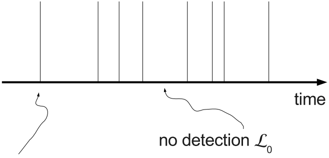
T. Krause, G. Schaller, TB, Phys. Rev. **B** 84, 195113 (2011).

- Electrons and phonons at different temperatures.
- 'Incomplete fluctuation theorem' with shift term $\rightsquigarrow$ current at zero bias.

See also O. Entin-Wohlman, Y. Imry, and A. Aharony, Phys. Rev. **B** 82, 115314 (2010).

Thermodynamics of Wiseman-Milburn Feedback

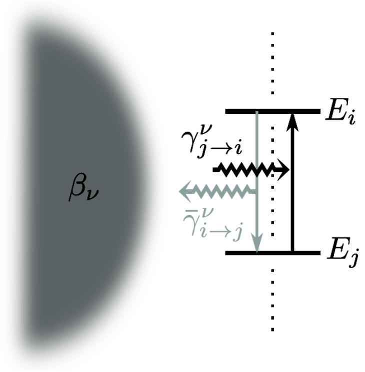
Feedback conditioned on quantum jumps



- **Scheme 1**
  - Rotate state vector by upgrading counting field χ in $\dot{\rho} = (\mathcal{L}_0 + e^{i\chi}\mathcal{J})\rho$ to superoperator $\mathcal{K}$, H. Wiseman, G. Milburn (1990s).
- Scheme 2
  - Effectively modulate system reservoir coupling $\rightsquigarrow$ Maxwell demon.

Thermodynamics of Wiseman-Milburn Feedback

Qubit model: absorption and emission of photons



- Master equation $\dot{\rho} = \mathcal{L}\rho$.
- Feedback-coupled populations and coherences

$$\mathcal{L} = \begin{pmatrix} \mathcal{L}_p & 0 \\ \mathcal{L}_{cp} & \mathcal{L}_c \end{pmatrix}$$