

Bose–Einstein Condensate – from superfluidity to superconductivity

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New approaches to the issue of superconductivity enable to treat Cooper pairs as bosons. In this meaning phase transition from the normal to the superconducting phase is treated as the Bose–Einstein Condensation (BEC).

Analogy between superconductivity and superfluidity is usually demonstrated with a ³He phase transition to the superfluid state where the two fermions have to create a boson to undergo the phase transition. The simplest model can be illustrated with the classical experiments which show the low temperature behavior of ⁴He. Presented experiments are a set of a few famous low temperature effects observed in ⁴He: λ transition, fountain effect and Kapitza's spider.

INTRODUCTION

Superfluidity is a phenomenon from the quantum physics field, where intuition is rarely enough to give the right description. It was discovered before quantum physics got fully developed, so first approaches to describe superfluidity had to base on the classical physics. Research on the liquid helium was possible after Heike Kamerlingh Onnes has liquefied it in the year 1908. The birth of quantum physics is dated back to the year 1900, when Max Planck presented his work on the blackbody radiation [1].

Comparison between the superconductivity and the superfluidity could be demonstrated with ³He phase transition to the superfluid state where two fermions (two ³He atoms) have to create a boson (pair of two ³He atoms) before the phase transition. Two electrons, as fermions, can also create boson. Such pairs are observed in a frame of the Micnas-Robaszkiewicz model [2] where temperature T_p is postulated. Phase diagram for superconducting materials start to be more complicated with an additional characteristic temperature T_p ($T_p > T_c$; T_c – critical temperature of the superconducting transition) which divide the region above T_c into two additional regions: Fermi-liquid above T_p (sometimes called Non-Fermi-liquid when the temperature is very close to T_p) and Bose-liquid in the temperature $T_p > T > T_c$ (sometimes called the pseudogap region). Region below T_c is treated as a BEC. T_p is the object of intensive experimental studies.

In the temperature region $T_p > T > T_c$ local pairs (Cooper pairs) are incoherent. During the cooling process local pairs can undergo Bose-Einstein condensation (BEC) or not, depending on the carrier concentration [3]. Coherent system of local pairs creates a superconducting state in case of high T_c

superconductors (HTS) [2] or a superfluid phase in case of ^3He . In both cases Bose-Einstein Condensation is the key phenomenon.

The simplest model of BEC is best illustrated basing on ^4He effects at the low temperature region.

SUPERFLUIDITY

To start a deliberation on the superfluidity it is necessary to move into the certain range of values of the basic thermodynamic quantities – the temperature T and pressure p . To do it man has to recall the phase diagram of the ^4He isotope naturally found on Earth in large quantities. It is impossible to solidify helium only by lowering T of the liquid helium if p is below 25 atm. First signals that there is some phase transition come from the electric permittivity ε measurements made by W. H. Keesom and M. Wolfke in the twenties of the last century. Figures 1 and 2 contain the $\varepsilon(p)$, $\varepsilon(T)$ and $\rho(T)$ (ρ – density) dependencies showing changes near the λ transition [4–9]. From these experiments it was possible to reproduce the phase diagram in the discussed p and T range. Phase diagram in the half-logarithmic scale is shown on the Figure 3 [4]. Helium becomes superfluid within the region marked with “He II”.

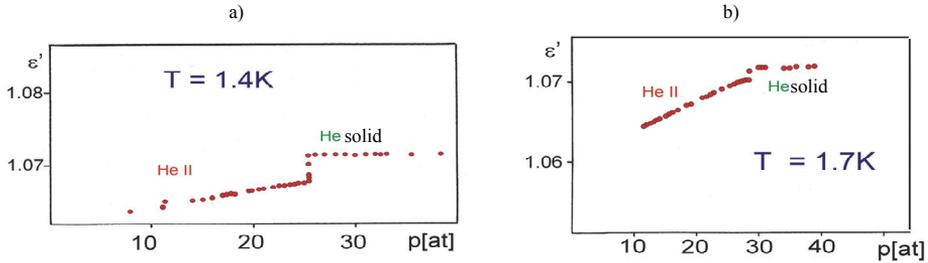


Figure 1. Real part of the electric permittivity of ^4He versus pressure in two temperatures below the λ point : a) 1.4K and b) 1.7K

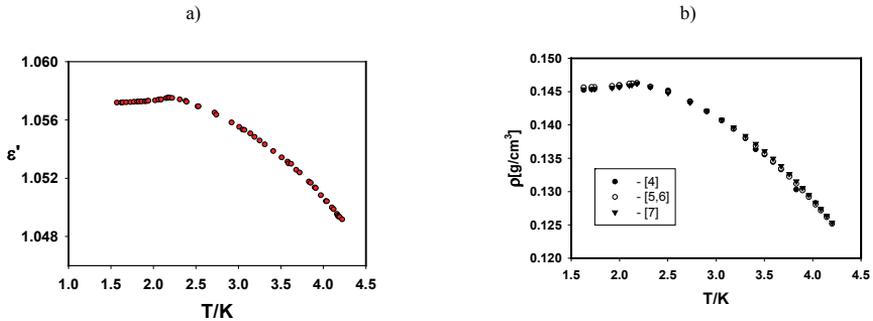


Figure 2 Permittivity (a) and density (b) of ^4He versus temperature close to the λ point

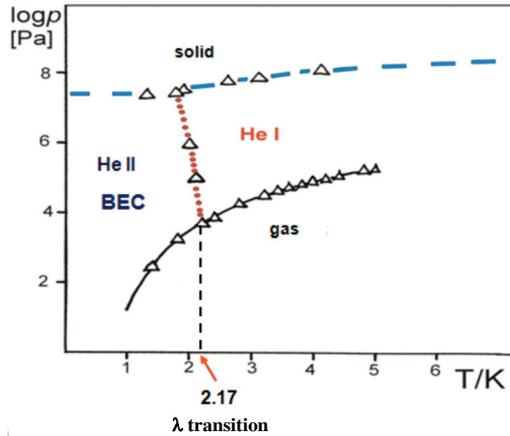


Figure 3. Phase diagram of ^4He plotted in the half-logarithmic scale basing on the results examples of which are shown on Figures 1 and 2

Existence of the solid phase of ^4He only in high pressures (see Figure 1) results from the high amplitude of the zero-point oscillations. Under the normal pressure this amplitude is much larger than the distance between the ^4He atoms; volume occupied by one atom is of the order of 46 \AA^3 [10]. The zero-point oscillations amplitude decreases with pressure faster than the distance between the atoms, thus the transition to the solid state becomes possible.

Interesting experiments with solid helium are still carried out. Two of them are especially worth mentioning: solidification of helium using the energy of an acoustic wave [11] and so-called “supersolidity” [12,13]. Both experiments are strictly connected with this article’s topic – the superfluidity. The first one shows the significance of the temperature for the creation of helium crystals. If the temperature is below the λ point the growth speed is amazing, for $15 \mu\text{m}$ crystals the observed growth time was 150 ns , what gives the speed of 150 m/s . Such high growth speed results from the extremely high thermal conductivity in the superfluid helium, which allows for effective transfer of the crystallization heat. High value of the thermal conductivity of the superfluid helium is the key to understand the experiment showing the λ transition, see Figure 4. The name of the effect observed in the second experiment – the “supersolid” helium – refers to the changes of the parameter being “equivalent” to the viscosity of a liquid – the elasticity. However the observed changes can result from existence of defects such as ^3He isotope addition rather than from the changes in the ^4He itself [14].

If it is impossible to solidify helium only by lowering the temperature, we should ask what happens during the cooling of the liquid helium. The process is easily accomplished by lowering the pressure over the liquid surface – removing of the high-energy atoms causes the system to lower its internal energy. Figure 3 shows this process with the curve separating the areas of “gas” and “He I” (classical liquid). When lowering the pressure, first we observe the strong boiling within the whole liquid helium volume (Figure 4a). Travelling along the “gas – He I” curve in the low temperatures direction we come across the specific point marked with “ λ ” at 2.17 K . At this point the volume boiling disappears (Figure 4b) and the helium surface becomes smooth despite the further decrease of the pressure (and the temperature as well) – the evaporation occurs only from the surface.

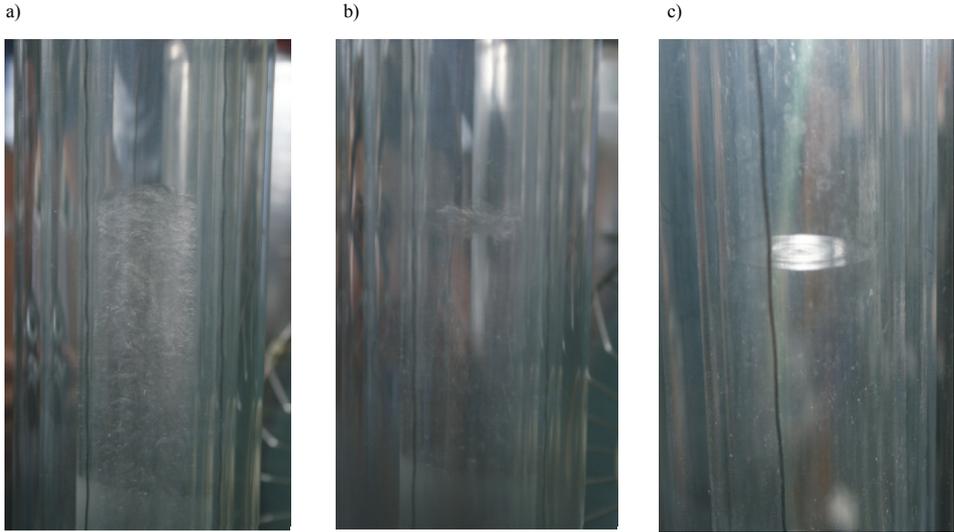


Figure 4. ${}^4\text{He}$ close to the λ transition: a) $T > T_\lambda$, b) $T \approx T_\lambda$, c) $T < T_\lambda$

The lack of volume boiling (no vapor bubbles inside the liquid) below the λ point means that there are no local overheating within the volume of He II. It means that it is impossible to create the stable temperature gradient within He II. The value of the thermal conductance above the λ point is of several orders of magnitude greater than the value below the λ point. He I is a classical liquid while He II shows the quantum behavior. The name of the transition comes from the Greek letter λ which is similar in shape to the temperature dependence of the specific heat of ${}^4\text{He}$. It was shown experimentally by W. H. Keesom in Leida at the beginning of the '30s of the past century. The term "superfluidity" appeared after the series of experiments performed by P. L. Kapitza [15,16], J. F. Allen and A. D. Missener [17,18]. They observed that ${}^4\text{He}$ below the λ point came through capillaries of 10^{-5} in diameter without any friction.

The first attempt to explain the He II behavior was so-called *two-fluid model*. It bases on the assumption that below the λ point, helium consists of two phases – the *normal phase* and the *superfluid phase*. Normal phase has all the properties of the classical liquid above the λ point, while the superfluid phase appears only below the λ point. The total density of the system is a sum of the two phases. The next assumption says that the superfluid phase has no viscosity, zero entropy and has very low energy. The superfluid phase plays a role of background for classical behavior of the normal phase. It is possible to separate the two phases with an *entropy filter* – the barrier for the classically-behaving normal phase. The superfluid phase can flow freely through it, as it has no viscosity. The third assumption of the two-fluid model says that the relation between the two component densities depends on the temperature. Low energy of the superfluid phase is strictly connected with the quantum state of the Bose-Einstein condensate. λ transition is also called as the *Bose-Einstein Condensation* [19] and the superfluid phase as the *quantum liquid*. The Bose-Einstein Condensation occurs for *bosons* (particles with integer spin), which occupy the lowest energy states. ${}^4\text{He}$ atoms (bosons) are able to occupy the same energy states, as opposed to the *fermions* (particles with half-integer spin) which follow the Pauli exclusion principle. Bose-Einstein Condensation is the condensation of bosons on the lowest energy level, which occurs in the momentum space.

The two-fluid model is a phenomenological approach and became a basis for the Landau theory, where basic excitations against the background superfluid phase are the phonons and rotons. R. Feynman

described rotons as “ghosts” of vortices appearing in the superfluid phase when the container with the liquid exceeds the critical rotational speed. Theories of Landau [20] and Feynman [21] base on the energy spectrum of helium, where the phononic and the rotonic branches can be identified.

Two-fluid model was confirmed in the famous Andronikashvili’s experiment [22]. Its recent version shows that the minimum number of helium atoms necessary to create a superfluid phase is sixty [23]. Two-fluid model nicely explains the *fountain effect* (Figure 5a) and the *Kapitza’s spider* (Figure 5b) which are *thermomechanical* effects generated with the heat delivered to the superfluid helium. The *mechanocaloric* effects, on the other hand, is the change of temperature caused by the mechanical operation (like escape of the superfluid phase through the entropy filter).

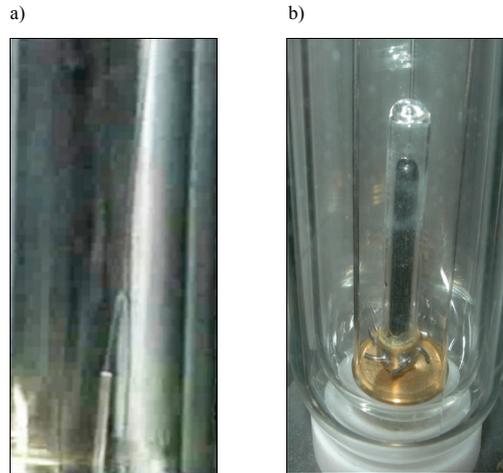


Figure 5. Fountain effect (a) and Kapitza’s spider (b)

Both effects are described with the London equation: $\Delta p = \rho S \Delta T$, where ρ is the total helium density and S is the entropy. London equation shows that existence of the local temperature gradients ΔT leads to generation of the pressure gradients Δp and vice versa. Such behavior explains the existence of the so-called *second sound*, which can be measured with a thermometer. The phase shift of the two independently oscillating phases results in a wave of temperature corresponding to the local changes in the density. The *third* and *fourth sounds* are observed in the superfluid helium membranes covering the walls of a helium container. The existence of a superfluid helium membranes creeping along the surfaces can lead to the emptying of a container if not protected properly [10].

CONCLUSION

To expose the similarities between the superfluidity and the superconductivity the simplest model of BEC was illustrated with the classical experiments showing the low temperature behavior of ^4He . Presented experiments are a set of a few famous low temperature effects observed in ^4He : λ transition, fountain effect and Kapitza’s spider.

The movie illustrating the effects mentioned above is available at the website of the ICEC23-ICMC2010 conference.

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