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Dynamics of ropes and chains

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Abstract

The main subject of the thesis is the analysis of the dynamics of ropes and chains based on data obtained from computer simulations.

In the first step of the study, a few rope models described in the available literature were analyzed. Based on this analysis, the equations of motion for the rope were formulated. For further research, the discrete model consisting of stiff segments connected by elastic–dissipative joints was chosen. In the most realistic model, the rope was seen as a system of stiff rods. In the simplest model, the rope was considered as a system of mathematical pendula. For the second model, it was possible to transform the equations of motion to a form more efficient from the point of view of numerical calculations. For both models, a series of tests using the available algorithms for numerical integration of differential equations were performed. Based on the analysis obtained, two algorithms: *DOPRI8* and *RADAU5*, were selected for further studies. The algorithms that were used depended on the type of considered problem (stiffness of the equations of motion).

The subject of further research was the analysis of the motion of the falling chain, initially folded. For this problem an approximate analytical solution was found, resulting from the principle of conservation of energy. This solution showed that the end of the chain fell with a grater acceleration than the gravitational acceleration. The analysis of the forces acting on the chain during its fall allowed the author to explain the mechanism behind the acceleration growth of the chain end. The analytical results were compared with results of laboratory experiments and numerical simulations.

A further stage of studies involved the problem of the fall of a chain where the initial distance between the ends of the chain was increasing. A number of laboratory and complementary numerical experiments were performed. As a result of this research, the dependencies of the velocity, the acceleration and the fall time of the chain end (depending on the initial distance between the chain ends) were presented. The analysis showed that for chains with a large initial distance between the ends (close to the chain length), the dynamics is different.

This case was investigated in details.

The next stage of research was to analyze the forces acting on a point of the support for the falling chain. For the case of the tightly folded chain, an approximate analytical solution was found. This solution showed that the forces acting on the support point can be very large and much larger than the weight of the chain. Results obtained from the simulations proved to be consistent with the analytical solution. Calculations were also carried out for the chain with a very large initial distance between its ends. Results of the calculations were compared with the experimental results obtained and were described in the literature. During this research an explanation was found for the observed experimentally (at the moment of time, at which the end of the chains was released) discontinuity in the value of the force acting on the support point of this chain. The discontinuity clearly appears when the distance between the ends of chain is large.

The next phase of the research was related to the analysis of the problem of the cracking whip. An approximate analytical explanation of the cracking sound emitted by a simple whip was presented and the relevant computer simulations were performed. Further research concerned the influence of parameters occurring in the equations of motion on the velocity of the whip end. As a result of this analysis, a simulation of the situation in which the velocity of the whip end exceeds the speed of sound was performed. In addition, the calculations corresponding to those described in the literature for the ”Australian cracking whip” technique were performed. For both techniques the explanation of the causes of the enormous speed and acceleration of the whip end was presented. The analysis of the dynamics of ropes and chains presented in this thesis is universal and it can be used in the analysis of other phenomena associated with this subject. The papers published during the work on the thesis were quoted in approximately fifty (see ”Cited by” section) papers regarding the dynamics of ropes and related fields.

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